1) Use Gaussian elimination to put the following system into *reduced* row echelon form. Use matrix notation. Find the solution set, using α notation (if necessary) in your answer.

$$x_1 + x_2 + x_3 = 1$$
$$2x_1 + 2x_2 + x_3 = 4$$

2) Write down the information in the traffic flow diagram on the board [The data was also typed up here. It is the HW exercise from Ch.1.2] as a 4x4 system of linear equations in standard form. You do not have to solve it.

3) Answer each part with "True" or "False". Explanations are not required.

   a) A 2x3 matrix in RREF must have at least two 1's .
   b) An overdetermined system cannot have free variables.
   c) Any two inconsistent 2x5 systems (involving the same variables) are equivalent.
   d) An underdetermined system can have a unique solution.
   e) A 4x4 linear system could have exactly four solutions.

Remarks: The questions counted 20 points each. The average, excluding a few low scores, was 47 / 60, almost 80%, which is good, but also fairly normal for a first quiz. The two high scores were 56 and 56. The scale below is advisory, probably a bit higher than the one on the syllabus because the grades are (for now) a bit high.

   A's 51 - 60
   B's 45 - 50
   C's 39 - 44
   D's 33 - 38

Answers: 1) Please imagine a vertical line between columns 3 and 4 of each matrix (I just don't know how to type that). Explain each step briefly, for example, my first step below subtracts 2 times row 1 from row 2.

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 1 & 4 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & -1 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{pmatrix}$$

The solution set is $S = \{3 - \alpha, \alpha, -2\}$.

There is no rule that you have to do exactly what I did, but there are GE rules. You cannot combine two steps to save time. You can replace row 2 by row 2 minus 2 times row 1, as I did, but you cannot replace row 1 by that (you would get the same solution set but you are not following the rules of GE). These distinctions are becoming important in Ch.1.5, as we learn to think of each GE step in terms of an elementary matrix E.

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2) Each intersection leads to an equation, based on IN = OUT. There is some flexibility. For example, the first equation below could be $-x_1 + x_2 = -50$ instead, and the order of the four equations could be changed.

$$x_1 - x_2 = 50$$
$$x_1 + x_4 = 870$$
$$x_2 - x_3 = -120$$
$$x_3 = 350$$

In standard form, the coefficients and variables should be in order (so x_1 is before x_2, etc) on the left side of each equation and the four constants should be on the right. You should circle your answer, especially if you have written other equations nearby which could create ambiguity. For that reason, it is probably better not to solve the system, but I did not deduct for that if you clearly started from the system above, or circled your answer.

3) FFTFF (see me, if needed)