

NAME

Leave any electronics such as phones or calculators at the front of the room. Show all your work and reasoning for maximum credit. If you continue your work on the back, etc, leave a note. You may ask about any ambiguous questions or for extra paper. Don't use your own, and if you use mine, hand it in with your exam.

1a) Set up the following word problem as a system of linear equations, written in matrix form. A numbered stick is perfectly balanced on the point $x = 0$ (so imagine that this point is the midpoint). It is also perfectly balanced after attaching three balls weighing x_1 , x_2 and x_3 pounds (resp) at locations -2, 1 and -1 (resp). It is also perfectly balanced when the locations are 1, 2 and -1 (resp).

1b) How many solutions does it have ? You do not have to actually solve it, but if you deduce the answer another way, then briefly explain your reasoning.

2) Label each system as underdetermined, overdetermined or square. Also, label each system as RREF or REF (but not RREF) or neither.

$$A = \left(\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 5 \\ 0 & 0 & 2 & 0 & 4 \end{array} \right), \quad B = \left(\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right), \quad C = \left(\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & 1 & 4 \end{array} \right)$$

A) [write 2 answers here]

B) [2 answers here]

C) [2 answers here]

3) Answer each part with "True" or "False".

- a) Any two inconsistent 2x5 systems (involving the same variables) are equivalent.
- b) Gaussian elimination can change an overdetermined system to underdetermined.
- c) Gaussian elimination can change an homogeneous system to nonhomogeneous.
- d) If $AB = AC$ and $A \neq O$ (the zero matrix), then $B = C$.
- e) If A and B are two matrices and the multiplication AB is possible, then BA is too.

Remarks and Answers: The 3 questions were worth 40, 30 and 30 points. Two people scored 100 out of 100. The average among the top 28 students was about 85, which is quite good, but not very unusual for a Quiz 1. The material may get a bit harder. Here is an approximate scale for this quiz. It is higher than the scale on the syllabus, but the final scale for the semester will not be.

A's 93 to 100
B's 85 to 92
C's 75 to 84
D's 65 to 74

1a)

$$(A | \mathbf{b}) = \left(\begin{array}{ccc|c} -2 & 1 & -1 & 0 \\ 1 & 2 & -1 & 0 \end{array} \right)$$

I also accepted answers in the equivalent form $A\mathbf{x} = \mathbf{0}$. During the quiz, I corrected a minor typo in one location.

1b) It has infinitely many solutions because it is homogeneous and under-determined. I did not give full credit for indefinite phrases like "it probably has infinitely . . ."

2) You did not have to explain these answers, but that is a good idea in general, especially in harder problems:

2a) under, neither (because $a_{23} = 2$).

2b) under, RREF. For RREF, it is not important that the system is inconsistent. You can also ignore the dividing line (it is not important that this is an augmented matrix, or even that it represents a linear system).

2c) square (because A is 2×2), REF.

3) TFFFF. I encourage you to ask me or our LA about any of these that are not 100% clear.