MAS 3105 Feb 2, 2012
Quiz 2 and Key Prof. S. Hudson

1) [10 points] One MHW problem uses a command; floor(10*rand(6)). Describe briefly what this means (what does MATLAB do with this? does it compute an integral ? a column vector ? what would change if you entered floor(1*rand(4)) instead?)

2) [30 points] Find the inverse of this matrix

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

3) [30 points] Answer True or False. You do not have to justify your answers. If a sentence *might be* false in some cases (like $x > 2$), consider it false. Assume all the matrices below are 3 x 3.

If A is row equivalent to a singular matrix B, then A is also singular.

The determinant of an elementary matrix can be any non-zero real number.

det $A + B =$ det $B + A$.

If A is row equivalent to I then A^T is row equivalent to I.

If every a_{ij} is an even integer, then det (A) is a multiple of 8 (such as 16, -8, 0).

There is a nonzero matrix A such that $A^2 = O$ (the zero matrix).

4) [30pts] Choose ONE of these to prove. Use words and sentences and standard methods to completely explain your reasoning and your formulas. Some of these are just parts of theorems we did in class. If so, you are NOT allowed to simply quote the theorem! You ARE allowed to use previous theorems, definitions and/or previous HW.

a) [HW from Ch 2.2] Show that if A and B are 3×3 matrices and $AB = I$, then $BA = I$.

b) Show that is A is row equivalent to I, then $Ax = b$ has a unique solution (you can assume the shapes are OK; basically you are supposed to provide 1-2 parts of the TFAE proof here).

c) A square matrix A is non-singular if and only if det $A = 0$.

Remarks and Answers: The average was 67 based on the best 18. The scores were normal-to-good on Problems 2 and 3, low on 1 (apparently some people have not started the MHW yet!), and slightly low on 4. With practice, the proofs should actually bring your grade *up*. The scale for Q2 is

A's 80 to 100
B's 70 to 79
C's 60 to 69
D's 50 to 59

As usual, this is an unofficial advisory scale. Also, pluses and minuses will be used later. Two typos below were corrected later, for (3) in 2014 and for (2) in 2019.

1) rand(6) produces a random 6 by 6 matrix (with $0 < a_{ij} < 1$), 10* multiplies by 10, floor() rounds off the entries to integers.

2) I suggest starting with $[A|I]$ which only takes 2 steps, but using the adjoint is also OK. A student asked in class about the *adjugate*. Apparently, it is what we call the adjoint. Some people prefer that, since the word adjoint is also used for other things (the conjugate-transpose, coming in Ch 6).

$$A^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

3) TTTTTT

4) You should walk into the Quiz ready to prove the listed theorems, making 4b) and 4c) easy! Otherwise 4a) is not hard either (HW). I wrote the grade for 4 in various places. If you don't see it quickly, look in the lower right corner of page 1 or page 2.

4a) Take det of both sides to show that A is nonsingular (the problem above refers to Ch 2.2, which was a hint to use det). Then, with A^{-1} to work with, it is easy to show $B = A^{-1}$ and finish.

4b) See the proof of TFAE (do 2 parts out of 3). First use row eq to prove A is nonsingular, then use A^{-1} to show $\mathbf{x}$ is unique.

4c) See the textbook.