MAS 3105 Jan 31, 2013

Quiz 2 Prof. S. Hudson

1) [30 points] Find the determinants of these matrices. Even if you can do the work in your head, include a remark about your reasoning, or some work. Label your answers clearly (det $A = \ldots$).

$$A = \begin{pmatrix} 4 & 4 & 4 \\ 4 & 4 & 4 \\ 4 & 4 & 4 \end{pmatrix} \qquad B = \begin{pmatrix} 4 & 4 & 4 & 1 \\ 4 & 4 & 4 & 0 \\ 4 & 4 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \qquad C = \begin{pmatrix} 17 & 2 & 300 \\ 17 & 1 & 100 \\ 0 & 1 & 200 \end{pmatrix}$$

2a) [40 points] Suppose $A \in R^{n \times n}$ and $A^2 = O$ (the zero matrix). Show that $I - A$ is nonsingular and that $(I - A)^{-1} = I + A$.

Your answer should include a fairly short calculation, along with some explanation of why you are doing it, and/or what you can conclude from it. You can choose 2b) instead, but then you will get at most 30 points, rather than 40.

2b) [30 points max] Do NOT do 2b AND 2a. Find a 3×3 matrix M so that

$$M \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 10 \\ 2 \\ 5 \end{pmatrix}$$

3) [30 points] Choose ONE of these to prove. You can answer on the back.

a) State and prove the TFAE theorem from Ch 1.5. You can skip the part about $b \Rightarrow c$.

b) Show that if $A, B \in R^{3\times 3}$ and B is singular, then AB is also singular.

Remarks and Answers: Many people did not follow instructions very well this time, answering both parts of 2), omitting the wrong part of 3a, or failing to leave notes about answers on scratch paper. Please be careful, so that you get the grade you deserve! The average score among the top half was 73 / 100, with high scores of 97 and 90. So, use the same scale as for Quiz 1 ($A-$'s start at 83, etc).

Note: I seem to have a technical problem with the Quiz 1 records. Please return your Quiz 1 to me asap to make sure you get credit for it.

1) 0, 16, 0. Explained shortcuts were OK. None of these require much work. Note that in C), rows 2 and 3 add to row 1 so that two type III's produce a row of zeroes. Soon (mid chapter 3), we will be able to explain this example faster, by saying the rows are *linearly dependent*.

2a) This is not hard (if you use the definition of *inverse*), but only 8 people chose it, and the results weren't very good. The proof is mainly this calculation:

$$(I-A)(I+A) = I^2 + IA - AI - A^2 = I$$

It should be easy to explain each step and to explain why it shows that $(I-A)^{-1} = I + A$ (left to you). Remark: it is not always true that $(B-A)(B+A) = B^2 - A^2$, as in high school algebra, unless $BA = AB$.

2b) This was probably easier than intended. There are many many good answers, and most people did OK with trial and error. For example,

$$M = \begin{pmatrix} 10 & 0 & 0 \\ 2 & 0 & 0 \\ 5 & 0 & 0 \end{pmatrix}$$

3) See the text for 3a. I expected you to reproduce the Ch.1.5 HW for 3b (using TFAE). But many people found a simpler proof using det's (since det $B = 0$, det $AB = 0$, too), which I accepted.