MAS 3105 Jan 31, 2014
Quiz 2, Key Prof. S. Hudson

For problems 1 and 2, let

$$A = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 2 & 2 & 5 \end{pmatrix}$$

1) Find an elementary matrix E so that EA is in REF [typo corrected on 9/16/19 - was "RREF"].

2) Compute the cofactor A_{21}.

3) Answer True or False. You do not have to justify your answers. Assume all the matrices below are 3 x 3 (so this A not the same as the one above).

If $AB = AC$ then $B = C$.

If $A\mathbf{x} = \mathbf{b}$ is consistent, then $\mathbf{b}$ is a linear combination of the columns $\mathbf{a_i}$.

If A is row equivalent to I then A^2 is row equivalent to I.

If det $U = 0$ and U is in REF, then U has a row of zeros.

If every a_{ij} is an odd number, then A is nonsingular.

4) Choose ONE of these to prove. Use words and sentences and standard methods to completely explain your reasoning and your formulas. Some of these are just parts of theorems we did in class. If so, you are NOT allowed to simply quote the theorem! You ARE allowed to use previous theorems, definitions and/or previous HW.

a) [HW from Ch 2] Show that if A and B are 3×3 matrices and $AB = I$, then $BA = I$.

b) A square matrix A is singular if and only if det $A = 0$. [typo corrected; changed nonsingular to singular]

Remarks and Answers: The average was about 69, based on the top 15 out 20. The highest scores were 100 and 93. The unofficial scale for this quiz is

A's 77 - 100
B's 67 - 76
C's 57 - 66
D's 47 - 56

If your average for Quizzes 1 and 2 is in the 50's or below, you are failing, and you should consider dropping the course. Theoretically, it is not too late to improve, but only with some major adjustments such as doing twice as many exercises, meeting with your LA an hour per week, or etc. The material tends to get harder after Quiz 2, though this remark probably should not affect your decision on dropping, since the grades will be scaled. Answers:

1) Apply a type-III to I:

$$E = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix}$$

2) -1.

3) FTTTF

4) See the textbook for 4b. Part 4a was HW and is fairly easy if you prove these steps in order (I am leaving the justifications to you for practice):

det $A \neq 0$ (there was a subtle hint in the problem ('Ch.2') that det might be useful)
A^{-1} exists
$B = A^{-1}$
$BA = I$

Some common mistakes were to make unjustified assumptions (that A is nonsingular, or symmetric, or 2×2, etc). You should walk into each quiz ready for proofs like these; ones advertised on the HW page, or directly from assigned HW. I sometimes also include unexpected ones, but they tend to be simpler.