MAS 3105 Sept 16, 2016
Quiz 2 Prof. S. Hudson

1) Find a 3x3 matrix A with at least two nonzero entries such that $A^2 = O$ (the zero matrix). If this is not possible, explain why not.

2) Write $\mathbf{v}$ as a linear combination of $\mathbf{u}$ and $\mathbf{w}$. For maximum credit, solve this using a reliable method (guessing the answer may only get partial credit).

$$\mathbf{v} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad \mathbf{u} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \mathbf{w} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

3) Choose ONE of these to prove. Explain thoroughly. You can always answer on the back, but leave a short note below, such as 'see back'.

a) If A is square and nonsingular, then $(A^T)^{-1} = (A^{-1})^T$. Use both parts of the definition of inverse.

b) Prove this part of the TFAE theorem: If A is row equivalent to I, then A is nonsingular.

c) If A is square and nonsingular then $\det A \neq 0$. You may use prior lemmas from Ch. 2, as done in class.

Remarks and Answers: This quiz was loosely based on Quiz 2 from Summer 2009. The new problem 1) is a bit harder, and maybe also 2), but 3) is a bit easier with the extra option. The average grade among the top half was 40 out of 60, which is low for Quiz 2. The two best scores were 60 and 59. Here is a rough scale for the quiz:

A's 48
B's 42
C's 36
D's 30

For a rough semester grade so far you can average the letter grades on your Quiz 1 and 2. It may be a bit more accurate to average the two numerical scores and use the scale below. It is pretty close to the scale on the syllabus, and the numbers will probably decrease a bit more. It is hard to predict the effect of the HW and MHW at this point, so I usually omit those for now.

A's 52
B's 46
C's 40
D's 34

1) One answer is $A = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. One student found another example with 9 nonzero

entries! There were many correct answers, but the average grade on this problem was low, perhaps because it was hard to get partial credit.

2) The answer should be written $\mathbf{v} = 3\mathbf{u} - \mathbf{w}$, or something very close to this. You can calculate the coefficients $x_1 = 3$ and $x_2 = -1$ by solving a 2x2 linear system.

3) Part a) was HW and parts b) and c) are in the text. Do not use a theorem from Ch.2 to prove a theorem from Ch.1 (your reasoning is very likely to be circular, since Ch.2 depends on Ch.1). Though you do not have to duplicate proofs exactly from the book or lectures, you do need to prepare before each quiz. Improvising usually doesn't turn out well.