MAS 3105 Sept 27, 2017
Quiz 2 Prof. S. Hudson

Let

$$M = \begin{pmatrix} 1 & 0 & 3 \\ 1 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

1) For the matrix above, compute M^{-1}.

2) Compute det M. Remember to show work.

3) Answer True or False. You do not have to justify your answers. Assume all the matrices below are 3 x 3.

If A^2 is row equivalent to I then A is row equivalent to I.

If every a_{ij} is either 1 or 2, then A is singular.

If A and B are symmetric and $AB = BA$, then AB is symmetric.

If A is singular then A^T is also singular.

If $ABA = BAB$ then $B = A$.

4) Choose ONE of these to prove. Explain thoroughly. You can answer on the back.

a) If A is square and nonsingular, then $(A^T)^{-1} = (A^{-1})^T$. Use both parts of the definition of inverse.

b) Prove this part of the TFAE theorem: If A is row equivalent to I, then A is nonsingular.

c) [Main Theorem] If A, B are square and A is nonsingular, then det $AB =$ det A det B.

Remarks, Scale and Answers: The average was 50 out of 60, which is very good. The two highest scores were 59 and 58. The advisory scale is a little higher than the one on the syllabus, but that will change for the next quizzes.

A's 53 to 60
B's 47 to 52
C's 41 to 46
D's 35 to 40

1) The standard method is to apply GE to $(M|I)$, but you could use adj M, or maybe E_k's.

$$M^{-1} = \begin{pmatrix} 1 & 0 & -3 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2) det $M = 1$.

3) TFTTF

4) See the text or lectures for b, c. Here is a proof of 4a, which was HW.

Proof: Since A is nonsingular, A^{-1} exists. So, $A^{-1}A = I$ and $AA^{-1} = I$ (by the definition of inverse). Taking transposes of both equations, and using the $(AB)^T$ formula, we get $A^T(A^{-1})^T = (A^{-1})^T A^T = I^T = I$. By the definition of inverse, this means $(A^{-1})^T$ and A^T are inverses of each other, so $(A^T)^{-1} = (A^{-1})^T$. Done.

Remarks: A common mistake was to use $(A^T)^{-1}$ in the calculations before you know it even exists (in my proof, this is not known until the very end). Another was to use determinants, but that is a Chapter 2 topic and this is a Chapter 1 problem. You might try to prove this in the style of trig identities, $(A^{-1})^T = \cdots = (A^T)^{-1}$ (where the dots represent a sequence of simplifications) but I don't see a way to do that for this example.