MAS 3105 Sept 19, 2019
Quiz 2 Prof. S. Hudson

Let

$$M = \begin{pmatrix} 1 & 0 & 3 \\ 1 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

1) For the matrix above, compute M^{-1}.

2) Compute det M. Remember to show work.

3) Answer True or False. You do not have to justify your answers. Assume all the matrices below are 3 x 3.

If A^2 is row equivalent to I then A is row equivalent to I.

If every a_{ij} is either 1 or 2, then A is singular.

If A and B are symmetric and $AB = BA$, then AB is symmetric.

If A is singular then A^T is also singular.

If $ABA = BAB$ then $B = A$.

4) Choose ONE of these to prove. Explain thoroughly. You can answer on the back.

a) If A is square and nonsingular, then $(A^T)^{-1} = (A^{-1})^T$. Use both parts of the definition of inverse.

b) Prove this part of the TFAE theorem: If A is row equivalent to I, then A is nonsingular.

c) [Main Theorem] If A, B are square and A is nonsingular, then det AB = det A det B.

Remarks, Scale and Answers: The average was approx 90, which is excellent, and six people scored 100 (maybe because, unfortunately, this quiz duplicated an older one). Again, the advisory scale is higher than the one on the syllabus, but that will change.

A's 93 to 100
B's 83 to 92
C's 73 to 82
D's 63 to 72

1) The standard method is to apply GE to $(M|I)$, but you could use adj M, or maybe E_k's.

$$M^{-1} = \begin{pmatrix} 1 & 0 & -3 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2) det $M = 1$.

3) TFTTF. Note that the third one is not obvious but it is directly from the HW and has a simple proof, $(AB)^T = B^T A^T = BA = AB$.

4) See the text or lectures for b, c. Here is a proof of 4a, which was HW.

Proof: Since A is nonsingular, A^{-1} exists. So, $A^{-1}A = I$ and $AA^{-1} = I$ (by the definition of inverse). Taking transposes of both equations, and using the $(AB)^T$ formula, we get $A^T(A^{-1})^T = (A^{-1})^T A^T = I^T = I$. By the definition of inverse, this means $(A^{-1})^T$ and A^T are inverses of each other, so $(A^T)^{-1} = (A^{-1})^T$. Done.

Remarks: Don't use determinants, because that is a Chapter 2 topic and this is a Chapter 1 problem (so your reasoning would probably be circular). It seems that people who studied these proofs did quite well.