

1) [20pt] a) Find $\text{adj}(A)$. b) Use that to find A^{-1} . Check your work.

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}$$

2) [20pt each] Choose TWO to prove. Mark them clearly. You can answer on the back.

a) Prove that if A is nonsingular and upper triangular, then $\text{adj}(A)$ is also upper triangular.

b) Suppose A and B are both $n \times n$. Prove that if $AB = I$, then A is nonsingular. Do not quote HW here.

c) Use induction to prove that if A is upper triangular, then $\det A = a_{11}a_{22} \dots a_{nn}$.

d) Thm 3.2.1: $\text{Span} \{v_1, v_2, \dots, v_n\}$ is a subspace of V . [use the def of subspace]

e) For every $m \times n$ matrix A , $N(A)$ is a subspace of R^n . [use the def of subspace]

Remarks and Answers: The average was approx 40/60. The scale is approx

A's = 49 to 60

B's = 43 to 48

C's = 37 to 42

D's = 33 to 36

Currently the Quiz averages are approx 52, 44, 40 [these fluctuate a little as people drop or change rank] with a combined average of 45. You can estimate your current semester grade by averaging your 3 scores and placing that on the scale above, with a 4 point adjustment [ex A's = 53 to 60]. If you have one very low quiz score, you might imagine replacing that by your MHW1 score [multiplied by 0.60]. That may be a little optimistic - for example, some future quiz score could be lower. If you are confused about your grade[s], please see me.

1) $\det A = 6$, so $A^{-1} = \frac{1}{6} \text{adj } A$ [below]. The instructions to 'check your answer' meant 'confirm that $AA^{-1} = I$ ' by doing the multiplication. You should do that anyway; it is quick and easy compared to finding $\text{adj } A$.

$$\text{adj } A = \begin{pmatrix} 6 & -3 & 1 \\ 0 & 3 & -1 \\ 0 & 0 & 2 \end{pmatrix}$$

2) Most of these are in the text, on my help pages, or explained on quiz keys. Some additional remarks:

a) I hoped you'd see the idea from your work in problem 1). When $i < j$ (in the upper triangle), M_{ij} has a zero on its main diagonal, so that $A_{ij} = 0$ (some more explanation is left to you).

b) The key idea is to use a *determinant*. This was part of a Ch2 HW problem (and Ch 2 is about determinants!). I graded it and posted the answer.

c) Include a basis step (the case $n=2$) and an induction step (which should include the induction hypothesis) ! Most people who used this standard proof structure did pretty well. Others didn't.

d) see text

e) see text or lectures

Proofs become more important the further you go in mathematics, or even just in this course. Practice as much as you can with the textbook examples and exercises; feel free to bring your practice work to me or Adrian for checking.

While induction is a fairly important type of proof, it will not be used as much after Ch 2 of this course, and it is not the easiest method to learn. So, if you are just starting with proofs, I'd suggest focusing on simpler examples, which depend mainly on the definitions (of *subspace*, for example). And, make a habit of learning the definitions!