

1) [40pt] Find a spanning set for  $N(A)$ . Write your answer in set notation (as usual).

$$A = \begin{pmatrix} 0 & 3 & 1 & 1 \\ 1 & 1 & 1 & 3 \end{pmatrix}$$

2) [30pt] Find the (2,3) entry of  $B^{-1}$  from the ratio of 2 determinants (hint: this was HW from the section on Cramer's Rule).

$$B = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 4 & 3 \\ 1 & 2 & 2 \end{pmatrix}$$

3) [30pt] Prove ONE: You can answer on the back. Small extra credit for (c), if done well.

a) Prove that the element  $\mathbf{0}$  in a vector space is unique.

b) If  $L = \{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k\} \subset V$ , a vector space, then  $\text{span}(L)$  is a subspace of  $V$ .

c) Use induction to prove that if  $A$  is upper triangular, then  $\det A = a_{11}a_{22} \dots a_{nn}$ .

**Remarks and Answers:** The average grade among the top 18 was 74 / 100, which is quite good for a Quiz 3. The scores were very diverse, with 6 grades over 90, and 6 grades under 40. The approx scale for Q3 is

A's 83-100

B's 73-82

C's 63-72

D's 53-62

Several people have asked me recently about their chances to pass the course. Here's a rough guide. Add your 3 quiz scores. If you did not hand in MHW1, deduct 30 points from that. If the result is at least 160 out of 300, you have a pretty good chance for a C (though you may need to work harder). If you have two good scores and one very bad score (which will surely be dropped) your odds are probably a little better than this guide suggests. I will give you another guide (probably more accurate) after Q4. Feel free to see me at any time about your grades.

1)  $L = \{[-2/3, -1/3, 1, 0]^T, [-8/3, -1/3, 0, 1]^T\}$ . There are many other acceptable answers, but this is the one you get from following conventions.

Notice that this final formula does *not* contain  $\alpha$  or  $+$  or  $N(A)$ .

2)  $-3/4$ . This is from the HW. In my opinion, the author intended for you to use Cramer's rule: You can start with  $BB^{-1} = I$  and by focusing only on the third column of both

sides, you get a linear system. The unknowns are the 3rd column entries of  $B^{-1}$ :

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 4 & 3 \\ 1 & 2 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Then, the answer comes from  $x_2 = \det(B_2)/\det B = -3/4$ . On the Quiz, most people used  $B^{-1} = \frac{1}{4} \text{adj } B$  instead. I gave full credit, because the entries of  $\text{adj } B$  are technically determinants (except for the  $\pm$ ), but I doubt this was the intention.

3) See the text, or me.