MAS 3105 Feb 14, 2013
Quiz III and Key Prof. S. Hudson

1) [20pt] This is a slight rephrasing of HW 3.1.11. Define + on the set of column vectors in R^2 as usual, but define scalar multiplication by $\alpha \circ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \alpha x_1 \\ x_2 \end{pmatrix}$. Is R^2 a vector space with these operations ? Briefly justify your answer.

2) [10pt] What do you type onto the command line in MATLAB, to generate a random 5x5 matrix ?

3) [10pt] Describe how to create a coding matrix A so that A^{-1} has no fractions.

4) [30pt] True-False. You can assume all the matrices are square in problems 4 and 5.

The set $S = \{[x_1, x_2]^T : 3x_1 + 5x_2 = 0\}$ is a subspace of R^2.

The set $L = \{[1, 2]^T, [3, 4]^T\}$ spans R^2 and is linearly independent.

For all scalars $\alpha > 0$, $\det(\alpha A) = \alpha$ (det A).

If $A\mathbf{x} = \mathbf{b}$ is consistent, then $\mathbf{b} \in$ span $\{\mathbf{a_j}\}$ (the columns of A).

If A is nonsingular, then A adj $(A) = I$.

5) [30pt] Prove ONE: You can answer on the back.

a) State and prove Cramer's Rule.

b) Use induction to prove that if A is upper triangular, then det $A = a_{11}a_{22} \ldots a_{nn}$.

c) If $L \subset V$ then span L is a subspace of V.

Remarks and Answers: The average among the top 15 was about 70, with a high score of 94 / 100. The unofficial scale is:

A's 80 - 100 B's 70 - 79 C's 60 - 69 D's 50 - 59

1) No. It fails Axiom 6 (you can give the formula or idea instead, of course).

2) rand(5)

3) Multiply a few type III matrices together, so that det $A = 1$.

4) TTFTF

5) See the textbook.