MAS 3105 Feb 14, 2014
Quiz 3 Prof. S. Hudson

1) Answer all three parts, based on the matrix A given below.

a) Find det A and det (A^{-1}).
b) Find adj A.
c) Use the answer to b) to find A^{-1}.

$$A = \begin{pmatrix} 3 & 1 & 5 \\ 2 & 0 & 4 \\ 0 & 0 & 3 \end{pmatrix}$$

2) Which of the following are subspaces of R^3? Explain each answer briefly.

a) $S = \{(x_1, x_2, x_3)^T |\ 3x_1 + x_3 = 0\}$
b) $S = \{(x_1, x_2, x_3)^T |\ x_1 x_3 = x_2\}$
c) $S = \{(x_1, x_2, x_3)^T |\ x_1 = 5x_2 = x_3\}$
d) $S = \{(x_1, x_2, x_3)^T |\ \ |x_1| + |x_2| = 1\}$

3) Choose ONE of these to prove on the back. Explain every step clearly.

a) If A is square, with 2 identical rows, then det $A = 0$. Use induction.
b) State and prove Cramer's Rule.
c) If $L_1 \subset L_2$ are two finite lists of vectors in V and L_2 is linearly independent (LI), then L_1 is also LI (use the definition of LI to prove this).

Remarks and Answers: The problems were worth 30, 40 and 30 points, and people did about equally well on all of them. The average was approx 69, with high scores of 97 and 93. The unofficial scale for this quiz is the same as Quiz 2:

A's 77 - 100
B's 67 - 76
C's 57 - 66
D's 47 - 56

I also computed the average of your three quiz scores so far - see the upper right corner of your quiz. You can use this to check that my records are correct and you can use the scale above with it. The average average is about 70 with a high of 99.

1) -6 and -1/6. A^{-1} is -1/6 times the adjoint, which is below:

$$\text{adj } A = \begin{pmatrix} 0 & -3 & 4 \\ -6 & 9 & -2 \\ 0 & 0 & -2 \end{pmatrix}$$

You really should check that $AA^{-1} = I$, which is easy to do. For this reason, I was a bit tough on partial credit for 1c).

2a) Yes (+6 pts). Because $S = N(A)$ for $A = [3\ 0\ 1]$ (+4 pts). Other explanations are possible, though it is hard to find another one this short. I gave 2 out of 4 points for incomplete / vague explanations such as simply quoting the definition of subspace.

2b) No. $[1\ 1\ 1] \in S$ but $3[1\ 1\ 1] \notin S$, so it's not closed under scalar multn.

2c) Yes (similar to 1a, except that now A is 2 by 3.)

2d) No. The zero vector does not belong.

3) See me or the text for answers or help. Also, I may have answered a and/or c on my Help pages, or on previous answer keys.