MAS 3105 — Sept 29, 2016

Quiz III Key — Prof. S. Hudson

1) [20 pt] Use Cramer's Rule to find the (2,3) entry of $B^{-1}$. Or, for partial credit, find the answer using some other method from Ch.2.3.

$$B = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 4 & 3 \\ 1 & 2 & 2 \end{pmatrix}$$

2) [15 pt] Find a spanning set for $N(A)$. Write your answer in set notation (as usual).

$$A = \begin{pmatrix} 2 & 1 & 3 & 4 \\ 1 & 0 & 1 & 2 \end{pmatrix}$$

3) [10 pt] Let $L = \{[1\ 2\ 2]^T,\ [5\ 3\ 3]^T,\ [6\ 5\ 5]^T\}$. Are these 3 column vectors linearly independent (LI) in $R^3$ ? Briefly justify your answer.

4) [15 pt] Prove ONE: You can answer on the back. Small extra credit for (c), if done well.

a) Prove that the element $\mathbf{0}$ in a vector space is unique.

b) If $L = \{\mathbf{v_1}, \mathbf{v_2}, \ldots \mathbf{v_k}\} \subset V$, a vector space, then span $(L)$ is a subspace of $V$.

c) Use induction to prove that if $A$ is upper triangular, then det $A = a_{11}a_{22}\ldots a_{nn}$.

**Remarks:** The average was 48 out of 60 (80%), with high scores of 60 and 56, which is pretty good. Here is the advisory scale for the Quiz:

A's 53 to 60
B's 47 to 52
C's 41 to 46
D's 35 to 40

Here is the advisory scale for your semester average (just the 3 quiz scores). It is possible that the lowest of your 3 scores will not count at the end, but if you only average your best two scores, don't use that average with this scale.

A's 51 to 60
B's 45 to 50
C's 39 to 44
D's 33 to 38

**Answers:**

1) Start with $BB^{-1} = I$, but focusing only on column 3 of both sides, we get $B\mathbf{x} = \mathbf{e_3}$. Then $x_2 = \frac{\det\ B_2}{\det\ B} = -3/4$.

But most people didn't use Cramer's Rule, and used $B^{-1} = \frac{1}{\det\ B}$ adj $B$. The (2,3) entry of adj $B$ is -3, which leads to the same answer, for 16 points credit.

2) The RREF is

$$U = \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

After using $\alpha$ and $\beta$ in the usual way, etc, I get $L = \{[-2\ 0\ 0\ 1]^T, [-1\ \ -1\ 1\ 0]^T\}$. There are many other possible answers.

3) Not LI. There were several acceptable explanations:

a) Combine into a matrix with det $A = 0$. It is singular, so the columns are LD.
b) As a shortcut, notice that $A$ has two identical rows.
c) Notice that $\mathbf{v_1} + \mathbf{v_2} - \mathbf{v_3} = \mathbf{0}$ (use the definition of LD).

4) See the text for 4b, 4c. See me for 4a. There were some pretty good induction proofs this time.