MAS 3105 Oct 13, 2017

Quiz III + Key Prof. S. Hudson

LI means linearly independent - you can always ask about such abbreviations.

1) Determine whether the following are subspaces of P_4, with explanation.

a) The set of polynomials of degree 3.

b) The set of polynomials in P_4 such that $p(0) = 0$.

2) Suppose $\mathbf{x_1}, \mathbf{x_2}$ and $\mathbf{x_3}$ are three LI vectors in R^3. Let $\mathbf{y_1} = \mathbf{x_1} - \mathbf{x_3}$, $\mathbf{y_2} = \mathbf{x_2} + \mathbf{x_3}$ and $\mathbf{y_3} = \mathbf{x_1} + \mathbf{x_2}$. Are $\mathbf{y_1}, \mathbf{y_2}$ and $\mathbf{y_3}$ also LI ? Prove your answer.

3) Prove ONE: You can answer on the back (but leave a note below).

a) Prove that the element $\mathbf{0}$ in a vector space is unique.

b) If $L = \{\mathbf{v_1}, \mathbf{v_2}, \dots \mathbf{v_k}\} \subset V$, a vector space, then span (L) is a subspace of V. For full credit, include all 4 steps.

c) State and prove Cramer's Rule.

Remarks and Answers: The average was 27 out of 60, with high scores of 49 and 35, which is unusually low. A possible explanation is that many people guessed wrong on 1a) and 2) and did not get any partial credit on those problems. Another is that the results often go down in Ch.3, which is fairly abstract, and requires more explanation than routine calculation. Here is an advisory scale for Quiz 3:

A's 35 to 60
B's 29 to 34
C's 23 to 28
D's 17 to 22

Here is an advisory scale for the semester so far, not yet including HW or MHW. Average your grades on the first 3 Quizzes and use that number here:

A's 44 to 60
B's 38 to 43
C's 32 to 37
D's 26 to 31

1a) No. It does not contain the zero vector (the constant function $p(x) \equiv 0$, which has degree zero, not three). Other explanations are possible.

Several people confused this set with P_3, which is a subspace. That seems understandable, and if the mistake was clear enough, I gave a little partial credit. Some people did not answer with a clear Yes or No.

1b) Yes. The shortest explanation I know is that $S = \text{span }(x^3, x^2, x)$ which must be a subspace. It is also OK to go over the 4-part definition of subspace. It is not OK to say S is a nullspace (which must contain column vectors, not polynomials).

2) No. Notice that $\mathbf{y_3} = \mathbf{y_1} + \mathbf{y_2}$ (if you suspect the list might be LD, you should hunt for equations like this). Since one is a LC of the others, the list is LD.

Or you could say $\mathbf{y_1} + \mathbf{y_2} - \mathbf{y_3} = \mathbf{0}$, which is a non-trivial LC, the definition of LD.

Several people tried to use matrices or linear systems for this problem, without much success. That is fairly difficult to do, since we do not know the entries of the $\mathbf{y_j}$.

Nobody tried an example, such as setting $\mathbf{x_j} = \mathbf{e_j}$. This does lead to specific entries of the $\mathbf{y_j}$ and to a singular matrix Y. You could conclude 'LD' (assuming that this example is typical). Of course, this is not a general proof, but it is better than answering 'LI', and it might get you onto the right track.

3) See the text and lecture notes. Option c) was most popular and the results were pretty good. A common mistake was to confuse the vector $\mathbf{x}$ with the entry x_i. Also, neglecting to explain each step.