MAS 3105 March 3, 2011
Quiz 4 and Key Prof. S. Hudson

1a) Let $\mathbf{u_1} = [1,2]^T$ and $\mathbf{u_2} = [0,1]^T$. Let $\mathbf{v_1} = [2,4]^T$ and $\mathbf{v_2} = [1,1]^T$. Find the transition matrix from $\{\mathbf{v_1}, \mathbf{v_2}\}$ to $\{\mathbf{u_1}, \mathbf{u_2}\}$.

1b) Use this to write $2\mathbf{v_1} + \mathbf{v_2}$ as a linear combination of $\mathbf{u_1}$ and $\mathbf{u_2}$.

2) True-False:

P_5 is a subspace of $C[0,1]$.

If A is a singular 3x3 matrix, then its nullity is at least 1.

If L is a list of n vectors, then dim (span L) = n.

$\emptyset$ is a subspace of $R^{2\times 3}$.

If L is a list of 3 L.I. functions in V, then dim $V \geq 3$.

3) Choose ONE (L.I. means linearly independent). You can answer on the back.

a) Prove this part of the 2/3 thm: If L spans V with exactly n vectors, and dim$V = n$, then L is L.I.

b) [3.3.2]]Let L be L.I. in V, and $\mathbf{v} \in$ span (L). Prove $\mathbf{v}$ can be written uniquely as a linear combination of vectors in L.

c) Suppose A is a nonsingular 3x3 matrix. Prove that its columns are L.I. in R^3.

Remarks and Answers: The average was approx 42 / 60. The scale is

A's 50-60
B's 44-49
C's 38-43
D's 32-37
F's 00-31

I estimated your semester grade by averaging your 4 quiz grades; see the upper right corner of your quiz. This estimate is probably accurate for students with no extreme scores. But it doesn't include your HW or MHW grades yet, and it may include a low quiz score that your MHW will replace later.

You can decide for yourself about whether to drop the course. As a general rule, most people will continue doing as they have done [barring some disaster, or a strong change of willpower, for example], with few changing by more than one letter grade after this point.

1) Set $M = U^{-1}V$, and then find $M\mathbf{x}$:

$$U^{-1}V = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix} \quad M\mathbf{x} = \begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$$

Conclude that the LC is $2\mathbf{v_1} + \mathbf{v_2} = 5\mathbf{u_1} + (-1)\mathbf{u_2}$. It's easy to check this; you should!

2) TTFFT; recall that part of the definition of subspace is that S cannot be empty.

3) See the text for a and b. For c, observe that $Ax = 0$ has only the trivial solution, which means there are no [non-trivial] dependency relations (but explain this in more detail for full credit).