MAS 3105 March 1, 2012
Quiz 4 and Key Prof. S. Hudson

1) Let $\mathbf{v_1} = (1,2)^T$ and $\mathbf{v_2} = (2,3)^T$ form a basis B for R^2. Find a pair of vectors $\mathbf{w_1}$ and $\mathbf{w_2}$ so that S is the transition matrix from $\mathbf{w_1}$ and $\mathbf{w_2}$ to B.

$$S = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

2) Answer True or False:

If the columns of A form a basis of R^4 then A must be 4x4 and nonsingular.

If $A \in R^{3\times 7}$ then $3 \leq \text{rank}\ (A) \leq 7$.

If $\mathbf{v} \in R(A)$ (the col space), then $\mathbf{v}$ is a linear combination of the columns of A in a unique way.

$L = \{1, 1+x, x+x^2, x^2+x^3, x^3\}$ is linearly dependent in P_7.

Every transition matrix is square.

3) Choose ONE proof. Answer on the back.

a) Two row equivalent matrices have the same row space.

b) $\text{rank}(A) + \text{null}(A) = n$

c) Suppose A and B are 3x3, and B is a nonsingular matrix. Prove A and BA have the same rank.

Remarks, Answers: The average grade among the top 16 was about 67%, which is normal. I don't usually compute averages for each problem, but did so this time. They were 73%, 66% and 58% resp. The (unofficial, advisory) scale for Q4 is

A's = 80 to 100
B's = 70 to 79
C's = 60 to 69
D's = 50 to 59

I have estimated your current semester grade on the upper right corner of your quiz, in blue ink. The number is the average of your best 3 quiz scores so far, out of 4. The letter is based on a scale like the one above, but about 5 points higher. This is probably more accurate than the estimate on Quiz 3 (which included all 3 quiz grades) but it does not include your MHW1 grade yet, nor the possibility that your lowest quiz grade might occur on Quiz 5 or 6.

In my experience, a very-hard-working student might improve by at most one letter grade from this point. For example, it is possible to go from a D to a C, with hard work, but it is not very likely you can go from a D- to a C. In past years, the numerical averages have gone down slightly from here, but the scales are set up so that the average letter grade should remain pretty stable. The eventual scale will probably resemble the one on the syllabus. If you have questions, or just don't understand this, you can see me.

1) $\mathbf{w_1}$ and $\mathbf{w_2}$ are the columns of W below. Start from $S = B^{-1}W$ (see examples from class or the text) so that $W = BS$. I gave decent partial credit if you started from a faulty formula such as $W = SB$, but only if your plan was clear. This is roughly exercise 3.6.7.

$$B = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} \quad W = BS = \begin{pmatrix} 7 & 10 \\ 11 & 16 \end{pmatrix}$$

2) TFFTT

3a) Most people chose this one, which was advertised [and in the text]. Usually the general idea was OK, but maybe not presented clearly, maybe with some mistakes in the vocabulary. You need a solid grip on the definitions of *row equivalent, linear combination,* and *row space* to write this proof well. Option 3b) is in the text, and 3c) was HW.