MAS 3105 Feb 28, 2013
Quiz 4 and Key Prof. S. Hudson

1) Let $\mathbf{u_1} = (1,1,1)^T$ and $\mathbf{u_2} = (1,2,2)^T$ and $\mathbf{u_3} = (2,3,4)^T$.

a) Write $\mathbf{x} = (3,2,5)^T$ as a linear combination of the $\mathbf{u_j}$. For maximal credit use a transition matrix.

b) Write $\mathbf{y} = (1,1,2)^T$ as a linear combination of the $\mathbf{u_j}$. Label your 2 answers clearly.

2) Let $S = \text{span}\ \{x^3 + x + 1,\ 2x^3 + x + 1,\ 3x^3 + x + 1,\ x + 1\} \subset P_4$.

a) Find a basis for S and briefly justify your answer.

b) What is the dimension of S ?

3) Choose ONE. You can answer on the back.

a) Prove this part of the 2/3 thm: If L is a L.I. set of n vectors in V, and $\dim V = n$, then L spans V.

b) Prove that any nonempty subset of a linearly independent set of vectors $\{\mathbf{v_1},\ \mathbf{v_2},\ \ldots \mathbf{v_n}\}$ is also linearly independent.

Remarks and Answers: The average of the top 15 scores was about 60 out of 100, which is a bit low, but not too unusual for Chapter 3. The high scores were 97 and 90. The unofficial scale for the quiz is

A's 73 - 100
B's 63 - 72
C's 53 - 62
D's 43 - 52

I have also estimated your semester grade in the upper right corner of your Quiz 4. This does not include your HW, MHW, your lowest quiz grade or any extra credit; just your best 3 out 4 quiz grades. To avoid mistakes - check this number ! The average of these scores is about 71, with highs of 94 and 88. I placed yours on a scale similar to the one above, but with A's from 80 - 100, etc, to estimate your letter grade.

1) [40 points] The t.m. is U^{-1} and the answers are a) $U^{-1}\,\mathbf{x}$ and b) $U^{-1}\,\mathbf{y}$, as given below:

$$U^{-1} = \begin{pmatrix} 2 & 0 & -1 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}, \quad U^{-1}\,\mathbf{x} = \begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix}_U, \quad U^{-1}\,\mathbf{y} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}_U$$

Ideally, you should take these a step further. For a), for example, you should write $\mathbf{x} = 1\mathbf{u_1} - 4\mathbf{u_2} + 3\mathbf{u_3}$ (and then you should check this is correct, because it so quick and easy to do so). But I gave full credit for the coordinate vector answer above.

I gave up to 35 points for solving the systems $U\mathbf{c} = \mathbf{x}$ and $U\mathbf{c} = \mathbf{y}$. But that method doesn't follow the instructions, and it takes longer (with more chances for mistakes).

2) [30 points] a) One basis is $L = \{x^3, x + 1\}$. It is clear that each element of the given spanning set is a LC of these two, so these also span S. Since they are not scalar multiples of each other, they are LI. b) dim $S = 2$, since we found a basis with exactly two vectors in it.

Many people wrote mysterious calculations with column vectors, which I just didn't understand, and I couldn't give credit for that. Column vectors make sense when you already *have* a basis to work with. It might be possible to use a basis for P_4, such as $B = \{x^3, x^2, x, 1\}$, to create a coordinate system and then use column vectors. But the stuff I saw was not like that.

3) See the text for 3a and the HW for 3b (which I will probably also go over in class asap).

If you are having trouble writing proofs, then *talk* to someone about it, for example, me or Camilo ! It is very helpful to learn what you are doing right and wrong this way. Reviewing graded work may not be enough.