MAS 3105 Oct 14, 2016
Quiz 4 Key Prof. S. Hudson

1) Which of the following are subspaces of R^3? Answer Y/N four times and explain each answer briefly, perhaps by writing S another way.

a) $S = \{(x_1, x_2, x_3)^T |\ x_1 + 4x_3 = 0\}$

b) $S = \{(x_1, x_2, x_3)^T |\ x_1 + 1 = (x_2 + 1)(x_3 + 1)\}$

c) $S = \{(x_1, x_2, x_3)^T |\ x_1 = x_2 = x_3\}$

d) $S = \{(x_1, x_2, x_3)^T |\ \ x_1 + x_2 = 3\}$

2) Let $\mathbf{u_1} = [1, 2]^T$ and $\mathbf{u_2} = [0, 1]^T$. Let $\mathbf{v_1} = [3, 4]^T$ and $\mathbf{v_2} = [0, 1]^T$. Find the transition matrix from the basis $\{\mathbf{v_1}, \mathbf{v_2}\}$ to the basis $\{\mathbf{u_1}, \mathbf{u_2}\}$.

3) True-False: no need to explain.

The polynomials of degree exactly 4 form a subspace of P_5.
If L_1 spans V, and $L_2 \subset V$ is L.I., and $L_1 \subset L_2$ then $L_1 = L_2$.
dim $R^{2\times3} = 6$.
There is a nonsingular 4x4 matrix A with null $(A) = 1$.
If A is a 3×5 matrix, then its column space has dimension ≤ 3.

Remarks and Answers: The average grade was 46 out of 60, with two high scores of 56 and 56, which is good, though I think this quiz was relatively easy. Here is a scale for it:

A's 51 to 60
B's 45 to 50
C's 39 to 44
D's 33 to 38

To estimate your semester grade is a bit awkward at this point since we do not know whether one of your current quiz grades will be replaced by your MHW. But let's assume that it will, and that your MHW average will be similar to your better quiz grades. So, average your best 3 out of 4 quiz grades so far, and use this scale. Note that computing your average this way gives 'inflated' results, so this scale is higher than the one on the syllabus, but it will come back down as more grades come in. If you are not doing the MHW you should probably average all 4 quiz grades. If you need help with any of this, see me.

A's 54 to 60
B's 48 to 53
C's 42 to 47
D's 36 to 41

1) The explanations below are the shortest I could find, and they mostly avoid discussing closure properties. But doing that is OK, and there are probably many other ways to

explain these. I did not require detailed proofs, but did require some fairly convincing logic and/or calculations.

1a) Yes. $S = N([1\ 0\ 4])$, so it must be a subspace.

1b) No. $[3, 1, 1]^T \in S$ but $[6, 2, 2]^T \notin S$. So, this is not closed under $\alpha\cdot$.

1c) Yes. $S = \text{span } \{[1\ 1\ 1]^T\}$, so it must be a subspace.

1d) No. It does not contain $\mathbf{0}$.

2) As usual in Ch.3.6, go from the V basis to standard and from there to the U basis.

$$T = U^{-1}V = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ -2 & 1 \end{pmatrix}$$

3) FTTFT