MAS 3105 Oct 27, 2017
Quiz 4 Prof. S. Hudson

1) The five vectors

$$\mathbf{x_1} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix},\ \mathbf{x_2} = \begin{pmatrix} 2 \\ 5 \\ 4 \end{pmatrix},\ \mathbf{x_3} = \begin{pmatrix} 2 \\ 7 \\ 4 \end{pmatrix},\ \mathbf{x_4} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix},\ \mathbf{x_5} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

span R^3. Pare this list down to a basis of R^3 and briefly justify your answer.

2) Let $L((x_1, x_2, x_3)^T) = (x_1, x_2, x_1)^T$ be a linear transformation of R^3. Let $T = \text{span}\ (\mathbf{e_2})$ and let $S = L^{-1}(T)$. Find a basis for S and briefly justify your answer. If you feel totally lost on this, you can find a basis for $U = \text{Ker}\ (L)$ instead, for partial credit. [Minor typo corrected 10/28/17: boldface removed from $\mathbf{x_1}$, etc]

3) Choose ONE. You can answer on the back.

a) Prove this part of the 2/3 thm: If L is a L.I. set of n vectors in V, and $\dim V = n$, then L spans V.

b) Prove: If $A, B \in R^{n \times n}$ and $AB = O$ (the zero matrix) then rank $(A)+$ rank $(B) \le n$.

c) Prove: If $AB = I$ and $BA = I$ then A, B are both square.

Remarks and Answers: The average among the top 18 students was approx 33 out of 60, which is a little low, but better than Q3. The high scores were 47, 46 and 45. I felt (without calculating) that the best average result was on problem 1 and the worst results were on problem 2. Here is an advisory scale for Quiz 4:

A's 38 to 60
B's 32 to 37
C's 26 to 31
D's 30 to 25

It is a bit unclear how to estimate your semester grade in any simple way, since we do not know yet whether one of your first four quizzes will be replaced by MHW, or maybe one of your next two will be. For simplicity, I suggest averaging your best 3 out of 4 quiz grades and using that number with the following scale. In effect, this assumes you have already had your lowest quiz grade, and that your HW and MHW averages will be fairly normal. You are welcome to discuss this with me. The DR deadline is Monday.

A's 47 to 60
B's 41 to 46
C's 35 to 40
D's 29 to 34

1) $B = \{\mathbf{x_1}, \mathbf{x_2}, \mathbf{x_5}\}$. Check that these are L.I. using a determinant (or etc). And there are exactly 3, so they form a basis.

Or, you can use x_5 and any two of the others. You cannot use three of $\{\mathbf{x_1}, \mathbf{x_2}, \mathbf{x_3}, \mathbf{x_4}\}$ because these four span only a 2-dimensional subspace. It might be good practice to justify these comments.

Most people seemed to find a basis quickly by trial and error, which was fine. A more methodical approach (but using Ch.3.5) is to create a 3 by 5 matrix A, and use GE to find a basis for Col (A).

2) $S = L^{-1}(T) =$ span $\{\mathbf{e_2}, \mathbf{e_3}\}$. It may be slightly better to write "$B = \{\mathbf{e_2}, \mathbf{e_3}\}$" - I accepted various styles. Reasoning: if $(x_1, x_2, x_1)^T$ is in T then $x_1 = 0$ (as occurs in both $\mathbf{e_2}, \mathbf{e_3}$). Ideally say more, about LI, for example. For partial credit: Ker $(L) =$ span $\{\mathbf{e_3}\}$. Reasoning: if $(x_1, x_2, x_1)^T = \mathbf{0}$ then $x_1 = x_2 = 0$ (as occurs in $\mathbf{e_3}$). Or you could get this answer from a routine nullspace calculation.

Most answers were not labeled, and not in the right form (eg a short list of vectors in R^3). I gave partial credit for plausible labeled answers.

3) Part (a) was advertised. See the text or lectures. Part (b) is a textbook exercise. Note that Col $(B) \subset N(A)$, consider the dims of both sides, and then use the rank+nullity theorem. Part (c) was the most popular choice. It was done in class using ranks, and I don't know of any other good method. Do not assume the two I's in the problem have the same size.