MAS 3105 Oct 17, 2019

Quiz 4 Key Prof. S. Hudson

1) [25pts] Let $S = \text{span } \{x^2,\ x^2 + 4x + 8,\ x + 2\} \subseteq P_3$. Find dim S. Explain.

2) [15 pts] Suppose $B = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$ is the transition matrix from a basis $\mathcal{B}$ of R^2 to the standard basis $\mathcal{S}$. Find $\mathcal{B}$ (list the vectors in order).

3) [15 pts] Give an example of a two-dimensional subspace of $R^{2\times 2}$.

4) [10 pts each] Is each $L : R^2 \to R^2$ a *linear* transformation? As always, explain.

4a) $L(\mathbf{x}) = \begin{pmatrix} x_2 \\ x_1 \end{pmatrix}$.

4b) $L(\mathbf{x}) = \mathbf{x} + \mathbf{e}_2$.

5) [25pts] Choose ONE. Assume A is square. LI means linearly independent. You can answer on the back. Remember - *words* !

a) Prove that dim Col A = rank A.

b) Prove that det A^T = det A.

c) Suppose A is a nonsingular 3x3 matrix. Prove that its columns are LI in R^3. Or, for partial credit, state clearly the definition of LI.

Remarks and Answers: The average on Quiz 4 was approx 50, with high scores of 91 and 87. This is a bit low. Here is an advisory scale for the Quiz:

A's 58 to 100
B's 49 to 57
C's 40 to 48
D's 30 to 39

The current average for the semester (based on the 4 quiz grades only) is approx 72, again with highs of 91 and 87. Here is an advisory scale for that average:

A's 80 to 100
B's 70 to 79
C's 60 to 69
D's 50 to 59

This semester scale is not entirely useful because your HW and MHW are not included. Their effects aren't easy to predict at this point. If we replace your lowest quiz grade so far by your MHW score so far, and do that for everybody, it might raise the previous scale about 5 points. Likewise, including the HW like a fifth quiz might raise the scale about 1 point. But I do not have enough data to be sure about these comments.

1) 2, because $S = \text{span } \{x^2,\ x + 2\}$, shows a basis with 2 vectors. Some additional

explanation is desirable, though I wasn't very picky about that. One key insight is that $x^2 + 4x + 8 = x^2 + 4(x + 2)$ is a LC of the other two. Also, that $\{x^2,\ x + 2\}$ is LI.

2) The two columns of B, because that's where we get the transition matrix.

3) Let $S = \text{span}\ \{M_1, M_2\}$, where M_1 and M_2 are almost any two matrices in the space. For example, let $M_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $M_2 \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$. These are LI because they are not scalar multiples of each other, so they form a basis with two elements, so dim $S = 2$. An equivalent answer is

$$S = \{\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} \mid a, b \in R\}$$

but for full credit get the notation right, and explain why dim S is 2.

Many people did not seem to realize this problem was about matrices and didn't get off to a good start. That is actually not a very important feature. If the vector space were P_7 instead of $R^{2\times 2}$ you could answer in a similar way, with $S = \text{span}\ \{x^3, x^5\}$ for example.

4a) Yes. Check the definition, or offer some similar reasoning, such as a matrix rep.

4b) No. $L(\mathbf{0}) \neq \mathbf{0}$.

5) See the textbook or lecture notes. Few people chose Part a, though the proof is mainly a fairly simple chain of equations. Part b should be done with induction, and is probably the hardest to explain. Part c is based on the TFAE theorem (that $A\mathbf{x} = \mathbf{0}$ implies $\mathbf{x} = \mathbf{0}$) and that in this context a dependency relation is a nonzero nullspace vector (so there are none).