MAS 3105 March 24, 2011
Quiz 5 and Key Prof. S. Hudson

1) Short Answers: a) [10pts; This is based on MHW 4.1]: Suppose $W = \text{triu}(\text{ones}(2))$, and we combine it's columns into a basis $F = \{\mathbf{w_1}, \mathbf{w_2}\}$. Suppose $L : R^2 \to R^2$ and $L(\mathbf{w_2}) = \mathbf{w_1}$ and $L(\mathbf{w_1}) = 2\mathbf{w_1} - \mathbf{w_2}$. Find the matrix representation of L with respect to F.

b) [5pts] What was the 2×2 matrix representation A in the rabbit story ? (recall each adult had two babies per year, which became adults in one year).

c) [5pts] A transition matrix can be thought of as a matrix representation of which simple linear transformation ?

2) Let $L : R^3 \to R^3$ be $L(\mathbf{x}) = [x_1, x_1, x_1]^T$. 2a) Find Ker(L).

2b) Find the range of L.

2c) Is L one-to-one ? (explain briefly)

2d) Is L onto R^3 ? (explain briefly)

3) Choose ONE of these.

a) Suppose $L : V \to W$ is linear. Show that ker (L) is a subspace of V.

b) Suppose that $A = ST$ and $B = TS$ where S is nonsingular. Prove that A is similar to B. [This should be pretty short, mainly a calculation, but include some words, such as the definition of *similar*].

c) Thm 3.6.6: Dim (Row (A)) = Dim (Col(A)).

Remarks and Answers: The Q5 average was about 37 / 60, rather low, but two students scored over 55 / 60. The scale for Q5 is:

A's 46 to 60
B's 40 to 45
C's 34 to 39
D's 28 to 33
F's 0 to 27

I've updated your estimated semester grade, based on this quiz and on dropping your lowest quiz grade so far. See the upper right corner, in blue ink. I have still not included HW or MHW into the estimate. Since this is a new method compared to the Quiz 4 estimate, there might be a few surprises, but most of the grades stayed about the same.

1a) and 1b)

$$M = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \quad \text{and} \quad A = \begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}$$

1c) the identity transformation

2a) $\{\mathbf{e_2}, \mathbf{e_3}\}$
2b) $\{\mathbf{e_1} + \mathbf{e_2} + \mathbf{e_3}\}$
2c) No; the Ker contains non-zero vectors.
2d) No; the range (see 2b) is not R^3.

3) See the text. I gave at least 15 points (usually much more) if the proof was logically correct (eg the steps could be justified), but only gave a perfect 20 if it included full explanations.