MAS 3105 March 22, 2012
Quiz 5 and Key Prof. S. Hudson

1) [30pts] Use the information below to compute the vector $\mathbf{x}_{100} \in R^2$ from the Rabbit example, done in class. Recall that $\mathbf{x}_0 = [1, 0]^T$ and $\mathbf{x}_n = A\mathbf{x}_{n-1}$. The eigenvectors of A were $\mathbf{v}_1 = [1, 1]^T$ and $\mathbf{v}_2 = [1, -2]^T$. You do not have to simplify.

$$A = \begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix} \qquad B^{-1} = \frac{1}{3}\begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}$$

2) [40pts] Let $E = \{1, x, x^2\}$ be a basis of P_3 and $F = \{1, x\}$ be a basis of P_2. Find a matrix representation A for the linear transformation $L : P_3 \to P_2$ defined by $L(p) = p'$ for these bases.

3) [30 pts] Circle ONE of these proofs, and answer on the back.

a) Let $L : V \to W$ be linear. Prove that Ker(L) is a subspace of V.

b) If Ker $(L) = \{\mathbf{0}\}$ then L is 1-1.

c) Thm 3.6.6: Dim (Row (A)) = Dim (Col(A)).

Bonus) Find a diagonal matrix D that is similar to the matrix A in Problem 1. If you answer on the back, leave a note here.

Remarks: The average on the Quiz was about 62, based on the top 16 scores, the lowest result this term. Apparently, Problem 1 was relatively difficult. The scale for **Quiz 5** is

A's 75-100
B's 65-74
C's 55-64
D's 45-54

Due to drops, and more grade data to work with, I have recalculated the quiz averages, based on the current top 16 students. The revised averages are 80, 64, 69, 64, 62. The average semester average is 74 (after dropping the lowest grade). I have estimated your current semester grade this way, and reported that to you in the upper right corner of your quiz. It does not yet include your MHW. If you have not handed in MHW, and have at least one low quiz grade, the estimate is probably too high. For a **semester** scale I am currently using

A's 84 -100
B's 74 - 83
C's 64 - 73
D's 54 - 63

Answers:

1) $\frac{2}{3}2^{100}\mathbf{v}_1 + \frac{1}{3}\mathbf{v}_2 = [\frac{2^{101}+1)}{3}, \frac{2^{101}-2)}{3}]$. I am posting a detailed review of the Rabbit example to show the work behind this answer. There should be a link very soon on your HW page and in my Help pages. I may ask about this example again on the final exam.

2) We did this in class.

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

3) See the text.

B) See the Rabbit pdf mentioned above.

$$D = \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$$