MAS 3105 Quiz 5 and Key Mar 21, 2013 Prof. S. Hudson

1) [40pts] True-False:

The 3×3 identity matrix I is rank deficient.

If $A \sim B$ then $A^T \sim B^T$.

If $A \sim B$ then N(A) = N(B).

If $A \sim B$ then rank $A = \operatorname{rank} B$.

With the notation of Ch 5.1 for projections, $|\alpha| = ||\mathbf{p}||$.

2) [30pts] (From the Rabbit Ex) Let $A = \begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\mathbf{x} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$. Compute either $A^{100}\mathbf{v}$ or $A^{100}\mathbf{x}$. One of these is hard, so pick the easy one! (and explain)

3) [30pts] Circle ONE of these, and prove it. Don't quote Thm 4.1.1 or past HW.

a) If $L: V \to W$ is linear, then L(V) is a subspace of W.

b) If Ker $(L) = \{0\}$ then L is 1-1.

c) If $A \in \mathbb{R}^{m \times n}$, and $B \in \mathbb{R}^{m \times m}$ is nonsingular, then rank $BA = \operatorname{rank} A$.

Remarks and Answers: The average (excluding one low score) was 57, with two high scores of 84 and 84. There were only a few grades in the B to C range this time. The unofficial scale is:

A's 68 to 100 B's 58 to 67 C's 48 to 57 D's 38 to 47

I have estimated your semester grade in the corner, like I did on Quiz 4, based on your best 4 quiz grades so far. The average of those averages is approx 67, which is a B^- .

1) FTFTT

2) $A^{100}\mathbf{x} = \mathbf{x}$ (because it is an eigenvector, with $A\mathbf{x} = -\mathbf{x}$ and $(-1)^{100} = 1$). The calculation of $A^{100}\mathbf{v}$ is much longer, but a couple of students got it from memory (see my link on the HW page to the Rabbit Ex for that answer).

3) See the text for 3a, and the others were HW.