MAS 3105 March 21, 2014
Quiz 5 and Key Prof. S. Hudson

1) These two matrices are row equivalent. U is in REF but not RREF.

$$A = \begin{pmatrix} 5 & 0 & 5 & 0 \\ 3 & 2 & 5 & 6 \\ 2 & 0 & 2 & 3 \end{pmatrix} \qquad U = \begin{pmatrix} 1 & 0 & 1 & 7 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

a) Find a dependency relation for the columns of A; give a nontrivial LC equal to the 0 vector.

b) Find a basis of $R(A)$.

c) Find a basis of $N(A)$ and find the nullity of A.

2) Find the vector projection of $[25, 0]^T$ onto $[3, 4]^T$.

3) Circle ONE of these proofs, and answer on the back.

a) Let $L : V \to W$ be linear, and let S be a subspace of V. Prove that $L(S)$ is a subspace of W.

b) If Ker $(L) = \{\mathbf{0}\}$ then L is 1-1.

c) State and prove Thm 5.1.1, the one with $\cos\theta$ in it.

Remarks: The average among the top 15 students was approx 58. The highs were 100 and 100 (again!). The scale is

A's 68 - 100
B's 58 - 67
C's 48 - 57
D's 38 - 47

I have written your semester average in the upper right, as usual. It is based on your best 4 out of 5 quiz scores so far. The average for this stat is 70, with highs of 100 and 98. The rough semester scale is

A's 79 - 100
B's 69 - 78
C's 59 - 68
D's 49 - 58

Answers:

1a) $\mathbf{a_1} + \mathbf{a_2} - \mathbf{a_3} + 0\mathbf{a_4} = \mathbf{0}$. I gave partial credit for other forms, such as a column vector. You can get to this from U, or from part c below.

1b) $\{\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_4}\}$ from looking at U. It is OK to write out the column vectors, of course. There are actually zillions of possible answers to this, such as $\{\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3}\}$, since $R(A) = R^3$, but for maximum credit you needed to show your reasoning.

A fairly common notational mistake was to write something like $R(A) = \{\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_4}\}$ (this needs the words *a basis of* or *span*).

1c) A basis is $\{[1, 1, -1, 0]^T\}$. The nullity is 1. This entire problem was also on Quiz 5 in 2009.

2) The standard formula gives $[9, 12]^T$.

3) See the text.