MAS 3105 Oct 28, 2016
Quiz 5 Prof. S. Hudson

1) [15 pts] Suppose that $L : R^2 \to R^2$ is linear and $L((1,2)^T) = (5,8)^T$ and $L((3,4)^T) = (9,25)^T$. Find $L((4,6)^T)$.

2) [15 pts] Finish the Rabbit example. The question is to compute $\mathbf{x}_{100}$. Here is enough of the work (you don't need to know the entries of A, etc): $\mathbf{x}_{n+1} = A\mathbf{x}_n$ and

$$\mathbf{x}_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{2}{3}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 1 \\ -2 \end{pmatrix}, \ A\begin{pmatrix} 1 \\ 1 \end{pmatrix} = 2\begin{pmatrix} 1 \\ 1 \end{pmatrix}, A\begin{pmatrix} 1 \\ -2 \end{pmatrix} = -\begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

3) [10 pts] One of your MHW exercises used the command $W = \mathbf{triu}(\mathbf{ones}(5))$. Show the output.

4) [20 pts] Circle ONE of these proofs, and answer on the back.

a) Thm 3.6.6: Dim (Row (A)) = Dim (Col(A)).

b) If A is similar to B then A^T is similar to B^T.

c) If A is similar to B then there are matrices S, T such that $A = ST$ and $B = TS$.

Bonus (about 5 pt): Given two matrices that can be multiplied both ways, must rank AB = rank BA ? Either give a proof or a counterexample.

Remarks and Answers: The average was 44, with high scores of 63 and 61, which is good. This average now includes all but approx 3 scores. The advisory scale is

A's 49 - 60
B's 43 - 48
C's 37 - 42
D's 31 - 36

For estimating your semester grade [assuming you are doing OK on the HW and MHW] average your best 4 out of 5 quiz grades and use the scale below. The class average for this is about 49, with a high of 61! I expect the average and the scale to naturally go down after Q6 and the final. See the scale on the syllabus for the highest possible final scale, in which A's would start at 85%, or 51 / 60.

A's 54 - 60
B's 48 - 53
C's 42 - 46
D's 36 - 40

1) $(14,33)^T$. Note $(1,2)^T + (3,4)^T = (4,6)^T$ and apply L to both sides (the problem is about linearity). In a harder example you might have to solve a system or find a matrix representation.

2) $(1/3)(2^{101}+1, 2^{101}-2)^T$. For the work, see the lecture notes or my help-links.

3)

$$W = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

4) The safe approach was to prepare for 4a) before the quiz. Otherwise 4b) is pretty easy; just write $A = S^{-1}BS$, take the transpose of both sides, etc, and explain a bit. Though part 4c) *should be* easy, many people confused the logic, trying to prove the converse, for example. Here is a proof of 4c):

Assume $A \sim B$, so by definition, $A = X^{-1}BX$ for some X. Let $S = X^{-1}$ and $T = BX$, so $A = ST$. Also, $TS = BXX^{-1} = B$. Done.

Remark: It is a little faster to start with $A = SBS^{-1}$ and $T = BS^{-1}$, which I accepted, but ideally this notation should be explained more, maybe using $B \sim A$.

Bonus) No. Here is an example such that $BA = O$ but $AB \neq O$:

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

This example also shows that the converse of 4c) is false. ST and TS cannot be similar unless they have the same rank, which does not always happen. But if one of the matrices is nonsingular, that changes everything.

It is not important in the bonus whether the matrices are square. But when discussing similarity we can always assume they are square, from the definition.