MAS 3105 Nov 8, 2017
Quiz 5 Key Prof. S. Hudson

1) Answer TRUE or FALSE. Assume the standard basis of R^2 is used.

a) For every matrix, $R(A)^\perp = N(A)$.

b) The matrix representation A of any rotation L in R^2 will have det $(A) = 1$.

c) If A is similar to I, then $A = I$.

d) The norm of the vector projection $\mathbf{p} = \text{proj}_{\mathbf{y}}(\mathbf{x})$ is the scalar projection α.

e) If A is similar to B and $A^2 = 4I$, then $B^2 = 4I$.

2) Define $L : P_3 \to P_2$ by $L(p) = p'(x) + p(1)$. Let $B = \{x^2, x, 1\}$ be a basis of P_3 and let $C = \{2, x+1\}$ be a basis of P_2. Find the matrix representation of L with respect to these bases.

3) Let $\mathbf{x} = (2, 4, 3)^T$ and $\mathbf{y} = (1, 1, 1)^T$. a) Compute the projection $\mathbf{p}$ of $\mathbf{x}$ onto $\mathbf{y}$.
b) Compute $\mathbf{x} - \mathbf{p}$ and check directly whether or not $\mathbf{x} - \mathbf{p} \perp \mathbf{p}$.

Bonus (about 5pts): State and prove one of the four theorems listed for this quiz.

Remarks and Answers: Unexplicably, the grading began based on 30 points per problem, so at the end I converted from a 90 point max to 60, to match the other quizzes, then added any bonus points. For example, 63/90 plus 5 bonus points became 42/60 + 5 = 47/60.

The average was 39/60 based on almost all the grades, which is fairly normal. The highest grades were 57 and 53. An advisory scale is

A's 44 to 60
B's 38 to 43
C's 32 to 37
D's 26 to 31

For a semester grade estimate, average out your best 4 out of 5 quiz scores and use the second Quiz 4 scale with B's starting at 41/60, etc. If your HW and MHW averages are abnormal, make some common sense adjustments. Each is worth approx the same as one quiz, but one with a relatively high class average. Or see me.

1) FTTFT. The fourth is a little tricky, $||\mathbf{p}|| = |\alpha|$, which is not always the same as α.

2) Start with $L(x^2) = 2x + 1 = -1/2(2) + 2(x + 1) = (-1/2, 2)^T$ in C coordinates, which gives the first column of A.

$$A = \begin{pmatrix} -1/2 & 1 & 1/2 \\ 2 & 0 & 0 \end{pmatrix}$$

3a) $\mathbf{p} = (3,3,3)^T$. It is OK to use α to get this, but people tended to make more mistakes that way.

3b) $\mathbf{x} - \mathbf{p} = (-1,1,0)^T$. Since $(\mathbf{x} - \mathbf{p})^T\mathbf{p} = (-1,1,0)(3,3,3)^T = 0$, they are orthogonal (and it is a theorem that they must be). The results were fairly low on this part, especially in the last step, due to various confusions and mistakes there - or earlier.

4) See the HW page for the list of 4 proofs. See the text or lectures for the proofs.