MAS 3105 | Oct 31, 2019
Quiz 5 | Prof. S. Hudson

Name

Show all your work and reasoning for maximum credit. You may ask for extra paper, but hand it back in with your exam.

1) [20 points] Suppose that $A \in R^{m \times n}$ has rank n and $A\mathbf{x} = \mathbf{0}$. Does this imply that $\mathbf{x} = \mathbf{0}$? Explain why, or give a counterexample.

2) [20 points] Suppose that $L\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a+b \\ 2a \end{pmatrix}$ is the transformation from the Rabbit example. Find the matrix representation M for the eigenvector basis, $\mathcal{B} = \{(1,\ 1)^T, (1,\ -2)^T\}$ of R^2, simplified. If you remember M, then also remember to show some work.

3) [30 points] a) Find the vector projection $\mathbf{p}$ of $\mathbf{x} = (1,1)^T$ onto $\mathbf{y} = (3,4)^T$.

3b) Find the vector projection $\mathbf{q}$ of the same $\mathbf{x}$ onto $\mathbf{z} = (-8,6)^T$.

3c) Show that $\mathbf{y} \perp \mathbf{z}$ with a calculation. What can you conclude about $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{x}$? How are they related ? Give a formula and justify it with words and a picture.

4) [30 points] True-False: The notation $A \sim B$ means the matrices are similar (Ch.4.3).

If $A \sim B$ then $A^3 \sim B^3$.

If $A \sim B$ then $R(A) = R(B)$.

If $A \sim B$ then $\det A = \det B$.

A 3×4 matrix in RREF with no non-zero rows must have full rank.

$L(A) = A - A^T$ is a linear transformation on $R^{n \times n}$.

Remarks + Scales: The average on the quiz among the top 23 scores was 54, with high scores of 77 and 71. Here is an advisory scale for the quiz:

A's 63 to 100
B's 52 to 62
C's 43 to 51
D's 34 to 42

To estimate your current semester grade, average your best 4 out 5 quiz grades and use the scale below. The average among the top 23 for that stat is approx 75. This system is not perfectly accurate, since it doesn't consider your HW and MHW grades, but those are not very definite yet. Including them would probably raise the scale slightly.

A's 82 to 100
B's 72 to 81
C's 62 to 71
D's 52 to 61

Answers:

1) Yes. Since rank = n, the columns are LI (this uses the "2/3 thm" of Ch.3). So there are no dependency relations among the columns. So, no non-trivial null space vectors.

You can also explain this quickly using the rank-nullity theorem.

2) $M = \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$. In this setting, M (or 'D') is always a diagonal matrix with the eigenvalues of L along the diagonal. The best method is probably the standard one from Ch.4, plugging in the basis vectors, etc. For example, the first column of M is

$$\mathbf{m}_1 = [L(\mathbf{b}_1)]_B = [L \begin{pmatrix} 1 \\ 1 \end{pmatrix}]_B = (\begin{pmatrix} 2 \\ 2 \end{pmatrix}_S)_B = \begin{pmatrix} 2 \\ 0 \end{pmatrix}_B$$

The subscripts B and S are a little awkward, but it is important to keep track of which coordinate system is being used, or should be used next. In the last step, we are converting from standard coordinates to B coordinates. Notice that $\begin{pmatrix} 2 \\ 2 \end{pmatrix} = 2\mathbf{b}_1 + 0\mathbf{b}_2$ which gives the B coordinates. This is easy to do here, but when it is not, we can use the transition matrix B^{-1}.

Another method is $M = B^{-1}AB =$ etc, but that takes longer.

3a) $\mathbf{p} = (21/25, 28/25)^T$.

3b) $\mathbf{q} = (4/25, -3/25)^T$.

3c) $\mathbf{y} \cdot \mathbf{z} = -24 + 24 = 0$, which shows $\mathbf{y} \perp \mathbf{z}$. But the main point of the problem is that $\mathbf{x} = \mathbf{p} + \mathbf{q}$. If you draw a careful picture of $\mathbf{p} + \mathbf{q}$ this should be clear (or ask me - nobody got this exactly right on the quiz).

4) TFTTT