1) Let $V = C[0, 2\pi]$ with inner product $\langle f, g \rangle = \frac{1}{\pi} \int_0^{2\pi} f(x)g(x) \, dx$.
   a) Show that 1 is orthogonal to $\cos(5x)$ in $V$.
   b) Compute $||\sin(3x)||$. [If you have forgotten some Calculus/Trig skills, you can replace $\sin(3x)$ by $x^2$, for partial credit].

2) Find the distance from the point $P(1,2,3)$ to the plane $x + y + 2z = 0$.

3) Choose ONE of these.
   a) [based on MHW 4.1] Suppose that $F = \{w_1, w_2, w_3\} = \{e_3, e_1 + e_2, e_1\}$ is a basis for $R^3$, and $L : R^3 \rightarrow R^3$. Suppose $L(w_1) = 4w_1$ and $L(w_2) = 4w_1 + 3w_2$ and $L(w_3) = 4w_1 + 3w_2 + 2w_3$. Find the matrix representation of $L$ with respect to $F$.
   b) Derive the formula for the projection matrix $P$ for a Least Squares problem (it represents projection onto $R(A)$). Then, show that $P^2 = P$.
   c) Show that if $\text{rank } (A) = n$ (= number of columns), then the normal equations have a unique solution $\hat{x}$. (You may use the textbook proof, or repeat HW 5.2.13, etc)

Remarks and Answers: The average was about 40/60. The scale for this quiz is: A’s start at 48, B’s at 41, C’s at 35, etc). I will post a revised semester scale ASAP. I expect it will be about 1-2 points lower than the one posted on the Quiz 5 key.

1a) $\langle 1, \cos(5x) \rangle = \frac{1}{\pi} \int_0^{2\pi} \cos(5x) \, dx = 0$

1b) $\langle \sin(3x), \sin(3x) \rangle = \frac{1}{\pi} \int_0^{2\pi} \sin^2(3x) \, dx = 1$ (and $1^{1/2} = 1$). Recall that $\sin^2(3x) = (1 - \cos(6x))/2$.

2) The scalar projection of $P$ onto $N$ is $\frac{9}{\sqrt{6}}$.

3a) 
   $$\begin{pmatrix}
   4 & 4 & 4 \\
   0 & 3 & 3 \\
   0 & 0 & 2
   \end{pmatrix}$$

3b) Get $P = A(A^T A)^{-1}A^T$ from the normal eqns. Then,
   $$P^2 = A(A^T A)^{-1} [(A^T A)(A^T A)^{-1}] A^T = A(A^T A)^{-1} A^T = P$$

3c) See the text or lectures. Of course, the main idea is to show $A^T A$ is nonsingular.