MAS 3105 April 4, 2013
Quiz 6 and Key Prof. S. Hudson

1) [30 pts] Let S be the subspace of R^4 spanned by $\mathbf{x} = (1,2,3,4)^T$ and $\mathbf{y} = (0,1,0,1)^T$. Find a basis of $S^\perp$. For a little EC [10 pts max], find an orthonormal basis of $S^\perp$.

2) [40 pts] a) Use the normal equations to find the Least Squares solution to the system $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 4 \\ 7 \\ 1 \\ 3 \end{pmatrix}.$$

2b) Find the projection vector $\mathbf{p} \in R(A)$ closest to $\mathbf{b}$.

2c) Something unusual happens in this example. For a little EC [5 pts max], identify and explain this.

3) [30 pts] Choose ONE of these to prove.

a) Thm 5.3.2: If A has rank n, then the normal equations have a unique solution. You may provide the textbook proof or the one from my lecture, based mainly on solving HW problem 5.2.13.

b) Derive the normal equations, used to solve least squares problems.

Remarks and Answers: The average grade was 69 / 100, with high scores of 107, 92 and 90. This is pretty good, though several people weren't prepared for the proofs. The unofficial scale is:

A's 79 - 100
B's 69 - 78
C's 59 - 68
D's 49 - 58

Again, I wrote your semester average in the corner (best 5 out of 6 quiz scores), which you should check. The average of these averages is approx 67. The scale is 4 points lower than the one above (so the A^-'s start at 75, etc). Your HW , MHW and final exam will be factored in later, of course. I also wrote in your lowest quiz grade, which will be replaced by your MHW average, just so you can check that my records agree with yours.

1) The results were good on this one. The most common answer was $\{[-3,0,1,0]^T, [-2,-1,0,1]^T\}$, from finding $N(A^T)$ through GE and removing the α, β as usual. Nobody got the full EC, which requires GSO (Ch.5.6).

2) $\hat{\mathbf{x}} = [1,3]^T$ and $\mathbf{p} = [4,7,1,3]^T$ which is $\mathbf{b}$. This happens because the original system was consistent, so that $\hat{\mathbf{x}}$ is not just a Least Squares solution, but a **true** solution to the system.

3) See the textbook or lecture notes. Most people chose a), probably because we went over this in class on Tuesday. Part b) is shorter, and can be based on a picture.