MAS 3105 Nov 13, 2016
Quiz 6 Prof. S. Hudson

1) Let $\mathbf{u}_1 = \frac{1}{3\sqrt{2}}[1,\ 1,\ -4]^T$, $\mathbf{u}_2 = \frac{1}{3}[2,\ 2,\ 1]^T$, $\mathbf{u}_3 = \frac{1}{\sqrt{2}}[1,\ -1,\ 0]^T$, an ortho. basis of R^3.

1a) Let $\mathbf{x} = [1\ 2\ 3]^T$. Write $\mathbf{x}$ as a linear combination of $\mathbf{u}_1$ $\mathbf{u}_2$ and $\mathbf{u}_3$ using Theorem 5.5.2. [find the coordinates of $\mathbf{x}$ w.r.t. this basis].

1b) Use 1a) and Parseval's formula to compute $||\mathbf{x}||$ (do not use standard coordinates).

2) Find the best least squares fit by a linear function to this data (so $f(1) \approx 5$ etc). Possible bonus for using Cramer's Rule to solve the normal equations.
$x = 1,\ 3,\ 4$
$y = 5,\ 4,\ 2$

3) Answer True or False.

If $\mathbf{x} = [1,\ -2,\ 5]^T$ then $||\mathbf{x}||_1 = 4$ (Manhattan distance).

If $\mathbf{x} = [1,\ -2,\ 5]^T$ then $||\mathbf{x}||_\infty = 5$.

The formula $\langle p, q\rangle = p(2)q(2) + p(3)q(3)$ defines an inner product in P_3.

Every 4x3 matrix A factors, $A = QR$, where R is nonsingular and $Q^TQ = I$.

If Q is 4x3 with orthonormal columns, then the matrix representation for projection onto $R(Q)$ is QQ^T.

Remarks and Answers: The average was 43 out of 60, with highs of 64 and 58. You can use this scale for the quiz:

A's 47 - 60
B's 41 - 46
C's 35 - 40
D's 29 - 34

For the semester, average your best 5 out of 6 quiz grades and use the scale below. I am assuming you are doing OK on the MHW, with an average of perhaps 80 out of 100, and that this (but out of 60) will replace your lowest quiz grade. The average average is currently 48 / 60 but I expect that to come down a bit after the final. The scale will come down too, to match the syllabus or perhaps lower. The highs for this stat are 62 and 55.

A's 52 - 60
B's 46 - 51
C's 40 - 45
D's 34 - 39

1a) $\mathbf{x} = \frac{-3}{\sqrt{2}}\mathbf{u}_1 + 3\mathbf{u}_2 - \frac{1}{\sqrt{2}}\mathbf{u}_3$. Use $c_1 = \langle \mathbf{x}, \mathbf{u}_1\rangle = \frac{-3}{\sqrt{2}}$, etc.

1b) $(c_1^2 + c_2^2 + c_3^2)^{1/2} = 14^{1/2}$. Use $c_1 = \frac{-3}{\sqrt{2}}$ etc, from 1a), not $c_1 = 1$ etc (standard coordinates), though both methods give the same numerical answer.

2) $f(x) = c_0 + c_1 x = 43/7 - 13x/14$. The normal equations are $\begin{pmatrix} 3 & 8 \\ 8 & 26 \end{pmatrix} \hat{\mathbf{c}} = \begin{pmatrix} 11 \\ 25 \end{pmatrix}$, which you can solve using GE, or with an inverse matrix, or by Cramer's Rule (which seemed cleanest to me).

3) FTFFT