MAS 3105 April 26, 2012
Final Exam Prof. S. Hudson

Just the problems. No answer key is available at this time.

1) (7pts) Solve for X, given that $AX + B = X$ and

$$A = \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix} \quad B = \begin{bmatrix} -1 & -2 \\ -3 & 0 \end{bmatrix}$$

2) (20 points) Answer True or False. Assume the matrices are 3x3.

If A is defective and similar to B, then B is defective.
If A and B have the same eigenvalues 4, 4 and 6, then they are similar.
If $\mathbf{x}$ is an eigenvector of A, then $2\mathbf{x}$ is an eigenvector of A^2.
If $\mathbf{x}$ is an eigenvector of A, then $\mathbf{x}$ is an eigenvector of $3A + I$.
If A is square and real then AA^T is diagonalizable.
If U is unitary then $U + I$ is unitary.
If U is unitary then U^T is unitary.
If U is unitary then U^H is normal.
If T is a Hermitian upper triangular matrix, then T is a diagonal matrix.
A and A^T have the same eigenvalues.

3) (20pts) Note: this problem may be easier than it looks.

a) Factor $A = QR$ (such that $Q^T Q = I$ and R is upper triangular), where

$$A = \begin{bmatrix} 4/5 & 0 \\ 0 & 1 \\ -3/5 & 0 \end{bmatrix}$$

b) Let $S = \mathrm{R}\,(A)$ and $\mathbf{w} = [1, 2, 3]^T$. Find the Least Squares solution for the system $A\mathbf{x} = \mathbf{w}$ and compute $A\hat{\mathbf{x}}$.

c) Find $\mathbf{p} = \mathrm{proj}_S(\mathbf{w})$ using the Ch 5.5 method (the summation formula). For a little extra credit, find the matrix representation for this projection operator.

d) Show that $\mathbf{w} \in S + S^\perp$ by splitting $\mathbf{w} = \mathbf{w}_1 + \mathbf{w}_2$, with $\mathbf{w}_1 \in S$ and $\mathbf{w}_2 \in S^\perp$.

4) (7 pts) Compute e^{3A} given that

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

5) (7 pts) Show that if A is nonsingular, then AA^T is also nonsingular. Be sure to include enough words of explanation.

6) (10pts) What does it mean to say that $\langle \mathbf{x}, \mathbf{y} \rangle$ is an *inner product*? State as much of the definition as you can remember. It has 3 main parts.

Suppose A is a nonsingular 4×4 matrix. Show that $\langle \mathbf{x}, \mathbf{y} \rangle = (A\mathbf{x}) \cdot (A\mathbf{y})$ is an inner product on R^4 (the dot in the formula is the usual dot product).

7) (5 pts) Use GE to get A into RREF;

$$A = \begin{pmatrix} i & 1 & i \\ 1 & 0 & 2 \end{pmatrix}$$

8) (14pts) a) Is the following matrix diagonalizable ? Justify your answer. Hint: one idea is to look at the rank of $A - 2I$.

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

8b) Find the eigenvalues of A^{-1}.

9) (10pts) Choose ONE:

A) If $\{\mathbf{x_1}, \mathbf{x_2}, \ldots \mathbf{x_k}\}$ are eigenvectors of A, from distinct eigenvalues, then they are L.I.

B) Prove that det $(AB) =$ det A det B. To save a little exam time, you can assume both matrices are nonsingular.

Bonus: (5pt) Suppose A is diagonalizable. Find a simple formula for $\det(e^A)$, such as $e^{\det(A)}$ or $e^{\mathrm{tr}(A)}$, and justify it.