3.1.3 - You need to prove the closure properties and the eight “axioms”. The closure properties are pretty obvious from the definitions in the exercise. The axioms usually require a calculation. For example, to prove A2, you can set \( x = a + bi \) (because that’s what it means to say \( x \) is in V). Likewise, set \( y = c + di \). Then calculate, like this (and justify briefly):

\[
\begin{align*}
x + y &= (a + bi) + (c + di) \\
&= (a + c) + (b + d)i \\
&= (c + a) + (d + b)i \\
&= (c + di) + (a + bi) \\
&= y + x
\end{align*}
\]

For A2, you’ll set \( z = (e+fi) \), and do a similar calculation. Etc.

3.1.7 - ”Unique” means ”there’s only one”. To show that \( 0 \) is unique, you assume that there are two vectors \( 0a \) and \( 0b \) that both satisfy axiom A3. Then the goal is to show that there really aren’t two different \( 0 \) vectors. That is, show that \( 0a = 0b \). Assumption A3 is that, for all \( x \) in V, \( x + 0a = x \) and \( x + 0b = x \). You can plug in \( 0a \) or \( 0b \) for \( x \), and get up to four useful equations from A3.

Now, can you combine some of these to prove the goal ?

3.2.10 - You can rephrase any of these to the question - ”is \( Ax = b \) always consistent (no matter what \( b \) is)?” You can always use GE, as in Example 11, page 140.

But there are usually shortcuts! If you have 3 vectors in \( R^3 \), you can use the determinant to decide if the matrix is nonsingular. If you have 4 vectors, you can probably throw one out, and still have 3 that span \( R^3 \) (just check that det is not zero), which means the 4 span it, too. If you have only 2 vectors, they won’t span \( R^3 \), but this is hard to explain quickly until later in Ch 3. For now, mimic Ex 11 (C). Be sure to explain your conclusions.

3.2.16 - Of course, \( R^1 = R \) is just the set of real numbers, and it is a very simple vector space. Vectors are just real numbers this time. This
exercise shows $R^1$ has only two subspaces. Of course, you do not have to prove the definition of subspace this time, because we are told that $S$ is one. You can assume that $0 \in S$ (but say it). There are two standard ways to prove an "or" conclusion, as we need to do in this exercise. The reasoning is pretty similar either way, but Option 1 is a little shorter.

Option 1: Assume $S$ is a subspace and that $S \neq \{0\}$ (which means there is some nonzero $v \in S$) and then prove $S = R$. To show that, we need to show every $x \in R$ must also be in $S$. Find a scalar $\alpha \in R$ such that $\alpha v = x$ (there is a simple formula for it). Why does that show $x \in S$?

Option 2: You can use cases instead. Then,

**Case 1:** Assume $S$ contains no nonzero numbers. [which means $S = \{0\}$ and ends this case].

**Case 2:** Assume $S$ contains a nonzero number $v$. [and show $S = R$, much like you did in Option 1].

3.2.18 - This is not too hard, but you need to know the definition of subspace well, and use it several times. Recall that $v \in U \cap V$ means $v \in U$ and $v \in V$. You need to show 4 things, starting with $0 \in U \cap V$ (showing the subspace is not empty). Etc.