FAQs on HW
Fall 2003

3.1.3 - You need to prove the closure properties and the eight "axioms". The closure properties are pretty obvious from the definitions in the exercise. The axioms usually require a calculation. For example, to prove A2, you can set **x** = a + bi (because that's what it means to say **x** is in V). Likewise, set **y** = c + di. Then calculate, like this (and justify briefly):

$$\begin{aligned}\mathbf{x}+\mathbf{y} &= (a+bi)+(c+di)\\ &= (a+c)+(b+d)i\\ &= (c+a)+(d+b)i\\ &= (c+di)+(a+bi)\\ &= \mathbf{y}+\mathbf{x}\end{aligned}$$

For A2, you'll set **z** = (e+fi), and do a similar calculation. Etc.

3.1.7 - "Unique" means "there's only one". To show that **0** is unique, you assume that there are two vectors **0a** and **0b** that both satisfy axiom A3. Then the goal is to show that there really aren't two *different* **0** vectors. That is, show that **0a** = **0b**. Assumption A3 is that, for all **x** in V, **x** + **0a** = **x** and **x** + **0b** = **x**. You can plug in **0a** or **0b** for **x**, and get up to four useful equations from A3.

Now, can you combine some of these to prove the goal ?

3.2.10 - You can rephrase any of these to the question - "is $A\mathbf{x} = \mathbf{b}$ always consistent (no matter what **b** is)?" You can always use GE, as in Example 11, page 140.

But there are usually shortcuts! If you have 3 vectors in R^3, you can use the determinant to decide if the matrix is nonsingular. If you have 4 vectors, you can probably throw one out, and still have 3 that span R^3 (just check that det is not zero), which means the 4 span it, too. If you have only 2 vectors, they *won't* span R^3, but this is hard to explain quickly until later in Ch 3. For now, mimic Ex 11 (C). Be sure to explain your conclusions.

3.2.16 - Of course, $R^1 = R$ is just the set of real numbers, and it is a very simple vector space. Vectors are just real numbers this time. This exercise shows R^1 has only two subspaces. Of course, you do not have to prove the definition of subspace this time, because we are *told* that S is one. You can assume that $0 \in S$ (but say it). There are two standard ways to prove an "or" conclusion, as we need to do in this exercise. The reasoning is pretty similar either way, but Option 1 is a little shorter.

Option 1: Assume S is a subspace and that $S \neq \{\mathbf{0}\}$ (which means there is some nonzero $v \in S$) and then prove $S = R$. To show that, we need to show every $x \in R$ must also be in S. Find a scalar $\alpha \in R$ such that $\alpha v = x$ (there is a simple formula for it). Why does that show $x \in S$?

Option 2: You can use cases instead. Then,

Case 1: Assume S contains no nonzero numbers. [which means $S = \{\mathbf{0}\}$ and ends this case].

Case 2: Assume S contains a nonzero number v. [and show $S = R$, much like you did in Option 1].

3.2.18 - This is not too hard, but you need to know the definition of *subspace* well, and use it several times. Recall that $v \in U \cap V$ means $v \in U$ and $v \in V$. You need to show 4 things, starting with $\mathbf{0} \in U \cap V$ (showing the subspace is not empty). Etc.