Linear Algebra, Hints on HW in Ch. 5.2
6/6/02

Most people can do HW 5.2.1 by imitating 5.2, Example 4. Some of the other HW in Chapter 5.2 is about bases of S and $S^\perp$. The key idea is to use a matrix A to do this. Usually you can choose A so that $S = R(A)$ and then $S^\perp = R(A)^\perp = N(A^T)$. So, you find $N(A^T)$ just like in problem 1. But each problem is a little different:

5.2.2 - Here $A = x$.

5.2.3 - Here, they choose A for you so that $S = R(A^T)$ (instead of $S = R(A)$ as I suggested above). Once you sort out 2a), you can use it to do 2b.

5.2.5a - This is very similar to the previous problems, once you decide what S is, and you find a spanning set for it.

5.2.10 - Good luck! This is an exercise in logic. It should only take about 2-3 sentences.

5.2.13 - See the other Hint page for this one.

Not assigned, but similar: 5.2.4 - Try it yourself, and then compare with my answer below:

Find a basis for $S^\perp$. This follows my outline above perfectly. Set $A = (\mathbf{x_1}, \mathbf{x_2})$ so that $S = R(A)$, and $S^\perp = R(A)^\perp = N(A^T)$. Finding a basis is now a Ch.1 problem. We solve $A^T\mathbf{x} = \mathbf{0}$ where,

$$A^T = \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & 3 & -2 \end{bmatrix}$$

This is already in RREF (Thank you, Dr. Leon!), so we set $x_4 = \alpha$ and $x_3 = \beta$, and get $x_2 = 2\alpha - 3\beta$ and $x_1 = -\alpha + 2\beta$. Factoring out the Greek, as usual, we find that the nullspace is spanned by $[-1\ 2\ 0\ 1]^T$ and $[2\ \ -3\ 1\ 0]^T$. These are LI, so they are a basis of $S^\perp$. Check that they are orthogonal to the basis vectors of S.