## Applications of topics in LA

When learning Linear Algebra the first time, it is often hard to imagine how some of the topics could ever be used in real life, or even in later chapters of Linear Algebra. The gray boxes in the book are devoted to applications, so look over each of those as you come to them. Chapters 2 and 3 have very few gray boxes, so you need some patience and faith then. Also, this page lists some major topics in Leon's book are how they are used later in the book or in non-math applications.

To keep it short, I've left out some common themes such as 'A is nonsingular' (used everywhere) and some ways of factoring a matrix (which can be used to improve some simple algorithms used in this course).
Chap Topic Application

1 G.Elim'n Solving systems (Ch.1), which appear in most sciences Finding $A^{-1}$ or $A^{-1} B$ (used often later!) Finding $N(A)($ Ch.3)

| REF + RREF | Part of the G.Elim process |
| :--- | :--- |
|  | Used in many proofs later (thm 2.2.2, 3.6.6, etc) |
|  | finding the row/column/null space of $A($ Ch3.6) |

E's
understanding G.Elim
factoring a nonsingular A (eg in Ch 2.2)
$A^{T} \quad$ converts rows to columns (See Ch3.5)
can use it to reverse order $\left(\operatorname{eg}(A B)^{T}=B^{T} A^{T}\right)$
a major tool in Ch5 (to do geometry)

2
$\operatorname{Det}(\mathrm{A}) \quad \begin{aligned} & \text { Testing if A is singular (Ch.2), can help find } A^{-1}(2.3) \\ & \\ & =\text { a ratio of areas (Multiv. Calc., the Jacobian matrix) } \\ & \\ & \text { Computing eigenvalues (Ch.6) }\end{aligned}$
3 vector space = 'the setting' for chapters 4-6, and much science subspaces
nullspace, $N(A)$ Computing eigenvectors (Ch.6)
Finding $S^{\perp}$ (Ch.5.2)
Finding $\operatorname{Ker}(\mathrm{L})(\mathrm{Ch} 4)$
linear indep. understanding basis, dimension, rank related to singularity of A
basis used to define/understand a new vector space a major idea throughout Chs 4-6

Projections Least Squares (approximations, Statistics, many sciences ) Used in the Gram-S process (Ch.5.6) Fourier series, Spectral Decomposition (6.4 HW19)

Inner Products Provides geometry when $V \neq R^{n}$ (eg Ch 5.7)
Gram-S method Used to get a better (ONL) basis of $S$ in many settings (eg Ch5.7, Ch6.4 Ex4, misc. physics).
To get $A=Q R$ (for least sq's (Ch5), or eigenvalues (7.6)
Eigenvalues Diagonalization of A (6.3)
Solving systems of DE's (6.2)
Diagonaliz'n To compute $A^{n}$ (for a Markov process) or $e^{A}$ (for DE's)
A is Hermitian $\begin{array}{ll}\text { Quantum physics, Pauli matrices } \\ & \text { Representation of self-adjoint operators }\end{array}$

