Lecture Notes in MAS 3105, Thursday 24, 2003

See Leon, Ch 1.4

After the MATLAB session, we went over a few final ideas about elementary matrices. Most of this is in the text, except maybe part 3).

1) Triangular factorization; read pages 74-75, especially the concluding paragraphs. The main idea is: given a matrix A, try to find triangular matrices L and U so that $A = LU$. How? Do GE on A, using only type III operations (using lower triangular E's), until U is upper triangular (if this is possible). Since products and inverses of lower triangulars are still lower triangular (proof omitted), we get the LU factorization, where L is lower triangular. We did not go into detail on why this is useful, but it helps in solving systems efficiently, and in designing circuits.

2) Read over the TFAE theorem (1.4.3) and its proof. We discussed the proof of each implication arrow of the "logic triangle", $a) \to b) \to c) \to a)$. The first and third arrows are pretty easy (see the text). The idea of the second one is this:

Reduce $Ax = 0$ to RREF, $Ux = 0$ (so, A and U are row equivalent). If b) is true, and the solution is unique, then there can be no free variables. So, there have to be n leading ones in the square nxn matrix U. The only way this can happen is if $U = I$, which proves a). This completes the proof of the arrows in the "triangle" and the proof of thm.1.4.3

3) GE works because it replaces a system by an *equivalent* system - it doesn't change the solution set. How do we know this? Suppose the original system is $Ax = b$. GE is the same as multiplying both sides of the system by a lot of E's, or by a single matrix M (the product of the E's). The new system is $MAx = Mb$. Clearly, any solution of the first system is a solution of the new one. Conversely, if x solves the new system, we can multiply that by M^{-1} (which exists, since the E's are all nonsingular) to see it solves the first system. So, the systems are equivalent.

End of lectures on Ch1. On Tuesday 1/29/03, we started Ch.2, Determinants.