Linear Algebra Final Exam, Review Sheet, last updated 4/20/11

The final is usually 2 hours (except in summers) in the usual room. For Spring 2011, it counts 30 points, but this may vary by semester (check your syllabus). About half will be on recent material (like a "Quiz 7") and about half will be review. It cannot be dropped; see the policy page about this, about incompletes, etc. It covers everything we will have done this semester, but it will emphasize:

a) Recent topics, as in 5.2 to 5.6, 6.1, 6.3 and 6.4 Omit Ch 5.7 and 6.2 [this refers to the 7th Edn of S.Leon's text].
b) HW 7, and practice problems beyond that (usually Ch.6.3-6.4). MHW3 may also be tested.
c) The proofs of thms. 2.2.3, 6.1.1, 6.3.1, 6.3.2 and 6.4.4. But if there is a different list on your HW page, it is probably more up-to-date than this; use that list instead.

Be able to do $S^{\perp}$, least squares, QR and diagonalization - and the practice exercises below. Expect problems slightly longer than on the quizzes.

Settle all business in the last week of class (late work, checking grades, regrading requests, excused absences from exams, etc). Any topics covered that week could be on the final, such as proofs from Ch. 6.4, spectral decomposition, idempotent/nilpotent matrices, simple Fourier polynomials, etc (but I do not include all of these every semester).

More Practice Exercises 4.3 - 8, 12, 5.2 - 15, 5.3 - 5, 5.4 - 5, 5.6 - 8

True - False Review: Assume that A is an arbitrary mxn matrix, that B is a square nxn matrix, and that C is a nonsingular square nxn matrix. Answers below (many of these are already on the TF practice quizzes on the web site exam page).

Rank (A) $\leq$ n and Rank (A) $\leq$ m.
Rank (B) = n
Rank (C) = n
Rank ($A^T A$) = Rank (A)
The columns of A are a basis of $R(A)$.

The columns of C are a basis of $R(C)$.

If the columns of B are independent then so are the rows of B.
If B is row equivalent to C then det (B) is nonzero.
If $m > n$ then the system $Ax = 0$ is inconsistent.
If $m < n$ then $Ax = 0$ has nontrivial solutions

The system $Cx = 0$ has a unique solution.
C is row equivalent to I.
B is a product of elementary matrices.
R(A) is a subspace of R^m.
N(A) is a subspace of R^m.

The dimensions of the row space and the column space of A are equal.
If the nullity of B is 1, then the rank of B is $n - 1$.
There is a set of 2n vectors that span R(C).
There is a set of n-1 vectors that span R(C).
If A represents a rotation of R^3 then A must be square.

If A is a transition matrix, then it must be square and nonsingular.
If B is similar to C, then rank (B) = n.
If C is an elementary matrix then so is C^{-1}.
det (BC) = det (CB)
det (B^H) = det (B)

If 3 vectors span R^3 they are a basis of R^3.
Any 4 vectors in P_3 must be dependent.
A set with only one vector must be independent.
A set including **0** must be dependent.
If $S \subseteq T$ are sets in V, then span $(S) \subseteq$ span (T).

Every 2x2 matrix has two eigenvalues.
If A has one eigenvector it has lots of them.
A singular matrix B has 0 as an eigenvalue.
Two similar matrices have the same eigenvalues.
Two similar matrices have the same eigenvectors.

Answers to True False:

TFTTF TTTFT TTFTF TTTFT TTTTF

TTFTT FTTTF