MAA 3200 Sept 22, 2003
Exam I Prof. S. Hudson

Name

If you continue your work on the back or another page, be sure to leave a note. Do not use a calculator, book, or any personal paper. You may ask me for extra paper, but hand it in with your exam. See me if there are any ambiguous questions (about the universal set, about missing quantifiers, etc). Each problem is worth 20 points.

1) Let A, B, C be arbitrary sets. Prove ONE of the following. Explain each step in words, but you don't have to explain any tautologies you use. Use complete sentences and the usual element-method wherever possible.

Prove a): If A and $B \backslash C$ are disjoint then $A \cap B \subseteq C$.

Or Prove b): If $A \cup B = B$ then $A \cap B = A$.

2) Answer TRUE or FALSE; you do not have to justify your answers.

$\neg(\exists x \in R, x < 1 \text{ and } x > 2)$

$\forall a, b, c \in Z$, if $a|c$ and $b|c$ then $ab|c$.

$\exists x \in R$, if $x^2 > 1$ then $x^2 < 0$.

$(p \vee q) \to r$ is logically equivalent to $p \to r \vee q \to r$.

$\exists! x, P(x)$ is equivalent to $\exists x(P(x) \wedge \forall y(P(y) \to y = x))$.

$\forall m \in N, \forall n \in N, \exists p \in N, mn|2p$.

If $mn|3$ then either $m|3$ or $n|3$ $(U = N)$.

$\forall x \in R[x \neq 2 \to \exists! y \in R, 2y/(y = 1) = x]$.

$\exists A, B, C$ such that $A \backslash B \subseteq C$ and $\neg(A \subseteq C)$ and $A \cap B = \emptyset$.

$\forall A, B, P(A \cup B) = P(A) \cup P(B)$. [$P$ stands for power set.]

3) Two of these are false. Circle them and disprove them by giving counterexamples, and some explanation.

a) $\forall a > 0, \forall b > 0, \exists c > 0, (c < a \wedge c < b)$ (where $U = R$).

b) $\forall x \in R, \forall z \in R, \exists y \in R, x^2 + y^2 = z^2$.

c) If $A \subseteq B$, $a \in A$, and a and b are not both elements of B then $b \notin B$.

d) If $|x - 3| < 2$ then $|x - 1| < 3$ or $|x - 7| < 3$.

4) Write out the precise definitions of each of these terms or phrases. Use logical notation like "$\forall$" whenever possible.

a) $A \subseteq B$

b) $\lim_{x \to a} f(x) = L$

c) $a|b$

d) $A \backslash B$

5) Let a, b, c be real numbers with $a > b$. Prove that if $ac \leq bc$ then $c \leq 0$. [You can use well-known facts about inequalities and positive numbers. If in doubt about this, see me!]