MAA 3200 Oct 24, 2013

Exam II and Key Prof. S. Hudson

1a) [10pts] Let $g : A \to B$ and $f : B \to C$. Prove that if $f \circ g : A \to C$ is onto, then f is onto.

1b) [10pts] Disprove the converse of the previous claim.

2) [5pts] Give an example of an anti-symmetric equivalence relation on N.

3a) [10pts] Let $S = N \times N$, and define a relation on S by $(a, b) \sim (c, d)$ iff $ad = bc$. Show that this is an equivalence relation.

3b) [5pts] List three elements of $[(3, 5)]$.

4) [10pts] Show that $(0, 1) \sim [0, 1]$ (same cardinality). You may use any theorems covered in class.

5) [10pts] State the Second Principle of Induction (often called Strong Induction).

6a) [7pts] Let $f : R \to R$ be defined by $f(x) = \lfloor x \rfloor$ (the floor function). Find $f(A)$, where

$$A = [3, 5] \cup (11, 15)$$

6b) [8pts] Find $f^{-1}(A)$

7a) [5pts] Define $Z/5 = \{0, 1, 2, 3, 4\}$ to be the usual congruence classes (equivalence classes) mod 5, as in Ch 4.5. Define + in the usual way so that $3 + 4 = 2$, for example. Is $(Z/5, +)$ a *group* ? Briefly justify your answer.

7b) [3pts] Define multiplication in the usual way, so that $3 \cdot 3 = 4$, for example. Does $(Z/5, \cdot)$ have an identity element ? If so, what is it ?

7c) [3pts] Which element, if any, is the inverse of 3 in $(Z/5, \cdot)$?

7d) [4pts] Is $(Z/5, \cdot)$ a group ? Explain.

8) [10pts] Choose ONE proof:

The power set of N is not countable.

If R is any relation on A then $R \cup R^{-1}$ is symmetric on A.

$\lim_{x \to 7} 7x^2 + 7 = 350$ (using the usual ϵ method).

Bonus: Give an example of a large set S such that there is no injection $f : S \to R$.

Remarks and Answers: The average (among the top 6 out 8) was 66, with highs of 78 and 71, which is fairly normal. The scores were mostly good except on problems 4, 5 and 7. This is a positive sign wrt basic skills, but you may need to pay more attention to major textbook theorems and definitions. The scale is

A's 73 - 100
B's 63 - 72
C's 53 - 62
D's 43 - 52

I have also averaged your two exam scores to estimate your semester grade (not yet including HW, EC or re-gradings) using the same scale as above. I wrote this in the upper right corner of your exam; please check it.

1a) Assume $g : A \to B$ and $f : B \to C$ and $f \circ g : A \to C$ is onto. ETS: f is onto. Let $c \in C$. ETS: $\exists b \in B$, $f(b) = c$. Since $f \circ g : A \to C$ is onto, $\exists a \in A$, $f(g(a)) = c$. So, $b = g(a)$ does it.

1b) We need an example so that f is onto and $f \circ g$ is not onto. Let $A = B = C = N$ and $f(n) = n$ and $g(n) = 2n$. Then $f \circ g = g$ which is not onto.

I hesitate to say that you MUST use an example here, but this is hard to explain clearly otherwise.

2) =

3a) People who got the notation right usually got the logic right too. I will prove symmetry here:

Assume $(a, b) \sim (c, d)$. ETS $(c, d) \sim (a, b)$. The first means $ad = bc$. The second means $cb = da$ which is the same thing.

3b) $[(3, 5)]$, $[(6, 10)]$, $[(30, 50)]$

4) The SB thm says: $A \preceq B$ and $B \preceq A$ imply $A \sim B$. The inclusion map shows $(0, 1) \preceq [0, 1]$. The 1-1 function $f(x) = x/2 + 0.01$ shows $[0, 1] \preceq (0, 1)$. Done.

The most common problem here was not knowing the SB Thm. In every advanced math class, you will be expected to know what the theorems say, especially theorems with names attached!

5) See the text. I did not expect you to memorize this, but we have used it several times. So, I expected you to remember the idea and write it out fairly precisely. Many answers seemed to have the right idea (maybe) but included nonsensical phrases such as "if n is true". Most people did not state an actual theorem, as requested, just a rough summary of how to use S.I.

6a) $\{3,4,5\} \cup \{11,12,13,14\}$

6b) $f^{-1}(A) = f^{-1}(\{3,4,5,12,13,14\}) = [3,6) \cup [12,15)$.

7) The results were a bit low on this one, not because it is truly hard, but it was from Chapter 8, a little ahead of the previous HW.

a) Yes. associative and has inverses.

b) [1], or you can just write 1.

c) 2

d) No, 0 has no multiplicative inverse.

8) 8a) is in the text and lectures, 8b) is an exercise, and examples like 8c) are in most Calculus texts. For 8b), I expected you to start with: Assume $(x, y) \in R \cup R^{-1}$ (etc). But one student found a more elegant proof using the theorem that $(R \cup S)^{-1} = R^{-1} \cup S^{-1}$.

Bonus: $P(R)$