MAA 3200 Dec. 14, 2000
Final Exam Prof. S. Hudson

Name

Show all your work and explain your reasoning. Don't use your own paper, a calculator or book. You may ask for extra paper or for clarifications. Assume the universal set is **R** unless noted. Most of the problems are worth 10 points each.

1) [20pts] Answer True or False: You don't have to explain.

The set of all polynomials is denumerable (countable).

If R and S are relations on A then $(R \cap S)^{-1} = (R)^{-1} \cap (S)^{-1}$.

If $f : A \to B$ then f^{-1} is a relation and its domain is B.

If $f : A \to B$ and B is a proper subset of A (so $B \neq A$) then f is not 1-1.

If $A \subseteq R$ and $f(x) = \chi_A(x)$, then $f^{-1}(\{1\}) = A$.

Z_5 is a field.

Z_8 is a field.

Every convergent sequence in R is bounded and Cauchy.

If $\liminf a_n = \limsup a_n = L$ then $\lim a_n = L$.

A finite set in a metric space M is always closed.

2a) Define the set P of positives in Q, using the usual [(p,q)] notation.

b) List the properties that P should have, for Q to be an ordered field.

c) Is Q an ordered field? (just answer yes or no)

3) Choose ONE:

A) If S is open in a metric space M, then S' is closed.

B) If $A \subseteq B$ in M then $\overline{A} \subseteq \overline{B}$.

4) Choose ONE:

A) Every Cauchy sequence converges.

B) Addition on R commutes (using definitions, facts about Q, and Cauchy sequences).

5) Choose ONE:

A) Prove or disprove: If $f : A \to B$ and $S, T \subseteq A$ then $f(S \cap T) = f(S) \cap f(T)$.

B) $\text{lub}(S \cup T) = \max(\text{lub } S, \text{ lub } T)$, assuming they exist.

C) $\lim a_n b_n = \lim a_n \lim b_n$, assuming they exist.

6) Prove that $\sum_{k=1}^{n} k(k+1) = n(n+1)(n+2)/3$ for $n \geq 1$, using induction.

7) (5 pts each) Give examples -

a) A set $S \subseteq R$ that is both open and closed.

b) A sequence with $\liminf a_n = 0$ that does not converge.

c) A set $S \subseteq Q$ that shows Q is not *complete*.

d) A counterexample to the Nested Interval Theorem if the I_n are allowed to be *open* intervals.

8) Let S be the relation $\{(x, y) : |x| \leq |y|\}$ on $\mathbf{R}$ (the reals). Give a definition of each word below, and state whether it applies to S. No proof required.

a) transitive (def? y/n?)

b) reflexive (def? y/n?)

c) symmetric (def? y/n?)

d) antisymmetric (def? y/n?)

e) partial order (def? y/n?)

BONUS [5pts]: Let $A = \{1, 2, 3, \ldots n\}$ and let $B = \{0, 1\}$. How many functions map A *onto* B?