MAA 3200 September 21, 2000
Exam I Prof. S. Hudson

Name

Show all your work and explain your reasoning. Don't use your own paper, a calculator or book. You may ask for extra paper or for clarifications. Assume the universal set is **R** except that in 3b and 5 it is $N = \{0, 1, 2, \ldots\}$.

1) [15pts] Use truth tables to determine which statements are tautologies, contradictions or neither. [Label each statement below].

a) $(p \wedge \neg q) \vee (p \to q)$

b) $(p \to q) \vee (p \to \neg q)$

c) $(p \wedge \neg q) \wedge (p \to q)$

2) [20pts] Prove that if $A \subseteq B - C$ and $A \neq \emptyset$ then $B \not\subseteq C$.

3) [15pts] One of these is false. Find it and disprove it by giving a counterexample.

a) $\forall a > 0, \forall b > 0, \exists c > 0, (c < a \wedge c < b)$ (where $U = R$).
b) If $ab|c$ and $ac|b$ then $a = 1$ and $b = c$ (where $U = N$).
c) If $A \subseteq B$, $a \in A$ and a and b are not both elements of B then $b \notin B$.

4) [15pts] Choose ONE to do:

A) Disprove $\lim_{x\to 2} 2x = 0$ using the definition.

B) Prove that $\exists x, (p(x) \vee q(x))$ is equivalent to $(\exists x, p(x)) \vee (\exists x, q(x))$. As in the HW, you may assume a similar equivalence involving $\forall$ and $\wedge$.

5)[15pts] Prove that x is even if and only if x^2 is even.

6) [20pts] Answer True or False: You don't have to explain.

$\forall x > 0, \exists! y > 0, x - 2y = 0$

If $A - C \subseteq B$ then $A - B \subseteq C$.

If $a < b$ and $ac \geq bc$ then $c < 0$.

$\exists! x((x-4)^2 = 9)$

$\forall x \geq 0, \exists y \geq 0, x + y = 0$