MAA 3200, Exam I Key Prof. S. Hudson

The average was about 70. You can use the original scale to find your approximate letter grade.

1) ANSWER: Taut/Taut/Contradiction

2) Prove that if $A \subseteq B - C$ and $A \neq \emptyset$ then $B \not\subseteq C$. *Brief Proof*: Since $A \neq \emptyset$, there is an $x \in A$ [optional details: $\neg(A = \emptyset)$ implies $\neg(\forall x, \neg(x \in A))$, which implies $\exists x, x \in A$]. So, $x \in B - C$. So, $x \in B$ and $x \notin C$. This shows that $\forall x, x \in B \rightarrow x \in C$ is false, and that $B \not\subseteq C$.

Note 1: It is *not* good to write "Let x be arbitrary" in this proof because we are not proving a "$\forall x, p(x)$" sentence here. The first sentence in the proof above introduces x the correct way (see 7, page 306).

Note 2: It is OK to assume $B \subseteq C$, to get a contradiction, but we still have to introduce x the same way as above.

3a) $\forall a > 0, \forall b > 0, \exists c > 0, (c < a \wedge c < b)$. True: pf (not required): given a and b, set $c = min(a, b)/2$.

3b) If $b|ac$ and $c|ab$ then $a = 1$ and $b = c$. This is false [ex: let $a = 0$, $b = 1$, $c = 2$].

3c) True [This is ex 3.2.5, page 102].

4) Choose ONE to do:

A) Disprove $\lim_{x \to 2} 2x = 0$. See the lecture notes for a similar ex: write out the definition, negate it; set $\epsilon = 4$ (or less), let $\delta > 0$ be arbitrary; let $x = 2 + \delta/2$; and check the '$P \wedge \neg Q$' part. Note: every formula should be introduced correctly with a word like "So," or "Assume" or "ETS" to explain its place in the proof. Pay attention to these words in the proofs you read (and write).

B) Prove that $\exists x, (p(x) \vee q(x))$ is equivalent to $(\exists x, p(x)) \vee (\exists x, q(x))$. See the key to the HW, where a list of equivalent statements is given. The brief explanation at the top is necessary - a sequence of formulas alone is not a proof! Do not write "Let x be arbitrary" in this one [see notes to (2) above].

5) This is ex 3.4.2, discussed on pages 122-124. Note: After writing "Assume x is even", you can say "This means $\exists k \in N, x = 2k$" (or say it in English), which introduces k into the proof. You should *not* write "Let k be arbitrary" (see the note to (2) again!).

6) Answer True or False:

$\forall x > 0, \exists! y > 0, x - 2y = 0$. True: $y = x/2$ is the unique solution.
If $A - C \subseteq B$ then $A - B \subseteq C$. True (try a proof!)
If $a < b$ and $ac \geq bc$ then $c < 0$. False: maybe $c = 0$.
$\exists! x((x - 4)^2 = 9)$. False: $x = 1$ or $x = 7$ (not unique).
$\forall x \geq 0, \exists y \geq 0, x + y = 0$. False: if $x = 1$ then $y = -1 < 0$.