MAA 3200 — October 19, 2000

Exam II — Prof. S. Hudson

Name

Show all your work and explain your reasoning. Don't use your own paper, a calculator or book. You may ask for extra paper or for clarifications. Assume the universal set is **R** unless noted.

1) [25pts?] Let $R = \{(a,a),(a,c),(b,b),(b,c),(c,c)\}$ be a relation on $A = \{a,b,c\}$.

a) Find the domain of R.

b) List the elements of R^{-1}.

c) List the elements of R^2.

d) Is R a partial order? (explain).

e) Is R an equivalence relation? (explain).

f) Is R a function? (explain).

2)[15pts] a) Show that $a \equiv b \pmod 5$ is an equivalence relation on Z.

b) How many equivalence classes are there in this example? Find at least 3 different equivalence classes.

3)[10pts] Prove or disprove: If $f : A \to B$ and $S, T \subseteq A$ then $f(S \cap T) = f(S) \cap f(T)$.

4) [15pts] Choose ONE:

a) Define Q from Z using ordered pairs and an equivalence relation. Explain briefly (but there's nothing to prove).

b) Suppose $f : A \to B$ and $g : B \to C$ are both onto functions. Then $f \circ g : A \to C$. Prove it is onto.

5) [20pts] Answer True or False: You don't have to explain.

The set of rationals, Q, is denumerable and $Q \sim Z$.

If R and S are relations on A then $(R \cap S)^{-1} = (R)^{-1} \cap (S)^{-1}$.

If $f : A \to B$ then f^{-1} is a relation and its domain is B.

If $f : A \to B$ and B is a proper subset of A (so $B \neq A$) then f is not 1-1.

If $A \subseteq R$ and $f(x) = \chi_A(x)$, then $f^{-1}(\{1\}) = A$.

6) [15pts] Choose ONE to prove:

a) Let R be a transitive relation on A. Show that $R^n \subseteq R$ for all integers, $n \geq 1$.

b) State and prove the Well-Ordering Principle using strong induction. [assume S has no smallest element and show it is empty].

c) Show that $\lim_{x \to 5} x^2 = 25$ using the definition (with δ etc).

BONUS [5pts?]: Let $A = \{1, 2, 3, \ldots n\}$ and let $B = \{0, 1\}$. How many functions map A *onto* B?