1) Let$R = \{(a,a),(a,c),(b,b),(b,c),(c,c)\}$ be a relation on $A = \{a,b,c\}$.

a) dom $R = A$.

b) $R^{-1} = \{(a,a),(c,a),(b,b),(c,b),(c,c)\}$.

c) $R^2 = R$ (this happens with transitive and reflexive Rs).

d) R is a partial order (it has R,T and A).

e) R is not an eq. relation (it fails S, since (a,c) belongs, but not (c,a)).

f) R is not a function (since (a,a) and (a,c) belong).

2) $a \equiv b \pmod 5$ is an equivalence relation on Z (done in lecture). There are 5 equivalence classes. For example, $[0] = \{\ldots -10, -5, 0, 5, 10, \ldots\}$. Also, [1], [2], [3] and [4] are OK (but $[5] = [0]$, etc).

3) FALSE - If $f : A \to B$ then $f(S \cap T) = f(S) \cap f(T)$. Look at examples where f is not 1-1 and $S \cap T$ is empty, such as: $f(x) = x^2$ with $S = (-1,0)$ and $T = (0,1)$. Note: *Many* plausible math statements are false - that's why we need proofs. If you guessed TRUE, you can prove "$\subseteq$", but not the other way (unless you assume f is 1-1; try it).

4a) Done in class (include a def of $(m,n) \sim (p,q)$) etc. 4b) Suppose $f : A \to B$ and $g : B \to C$ are both onto functions. Prove $g \circ f$ is onto. Pf: ETS $\forall c \in C, \exists a \in A, g \circ f(a) = c$. Let $c \in C$. Since g is onto $\exists b \in B, g(b) = c$. Since f is onto, $\exists a \in A, f(a) = b$. So, $g(f(a)) = g(b) = c$.

Note on 4b: People who didn't write out the ETS got confused here, and most never correctly introduced the variables. You can't use the fact that f is onto *until b has been introduced.* So, talk about c first (react to the ETS), then g and b, then f and a.

5) T - The set of rationals, Q, is denumerable and $Q \sim Z$.

T- If R and S are relations on A then $(R \cap S)^{-1} = (R)^{-1} \cap (S)^{-1}$.

F - If $f : A \to B$ then f^{-1} is a relation and its domain is B. (dom $(f^{-1}) \subseteq B$).

F - If $f : A \to B$ and $B \subset A$ then f is not 1-1. (ex: $f(n) = n + 1$, $f : N \to Z^+$).

T - If $A \subseteq R$ and $f(x) = \chi_A(x)$, then $f^{-1}(\{1\}) = A$.

6ab) Both done in class. 6c) Show that $\lim_{x \to 5} x^2 = 25$. See Anton's *Calculus*, for similar exs. Here's a brief pf: Let $\epsilon > 0$. Set $\delta = \min\ (\epsilon/100, 1)$. Assume $0 < |x - 5| < \delta$, so that $|x - 5| < \epsilon/100$ **and** $|x - 5| < 1$. Since $4 < x < 6$ we get $|x + 5| = x + 5 < 11$. So, $|x^2 - 25| < 11 \cdot \epsilon/100 < \epsilon$. Done.

Note: It is wise to use "min" in choosing δ because it gives us **two** inequalities for $|x - a|$. We need both, because $|x^2 - 25|$ factors into two parts. Practice more of these with x^2, $1/x$ etc.

BONUS: Let $A = \{1, 2, 3, \ldots n\}$ and let $B = \{0, 1\}$. There are 2^n functions $f : A \to B$, and only 2 (the constant functions) are not onto. So, $2^n - 2$.