MAA 3200 November 14, 2000
Exam III Prof. S. Hudson

Name

Show all your work and explain your reasoning. Don't use your own paper, a calculator or book. You may ask for extra paper or for clarifications. Each problem is 20 points.

1) Short Answer:

Give an example of a number system with additive inverses that is not a field.

Given that $s_1 = 1$ and $s_2 = 1$ and $s_{n+1} = s_n + s_{n-1}$ (for $n \geq 2$), find s_8.

What is the *trichotomy* property in an ordered field?

Give an example of a sequence that is not bounded, with $\lim s_n/n = 0$.

2) Answer True or False; do not justify.

In every field, $0 < 1$.

Z_5 is a field.

Q is a complete ordered field.

$\forall x, y \in N,\ s_x(\sigma(y)) = \sigma(s_x(y))$.

$s_2 : N \to N$ is onto.

Let $U = Q$. If $\{s_n\}$ converges, then it is Cauchy.

The set of all polynomial functions with rational coefficients is countable.

3,4) Choose TWO to prove:

A) Prove that multiplication is commutative in Z. Use the definitions - you may assume anything you need about N.

B) Addition in N is associative. Use induction.

C) $<$ is transitive in N (use the definition of $<$ and basic properties of $+$ and/or $\cdot$).

D) Suppose that $\{s_n\}$ is a Cauchy sequence with a subsequence that converges to L. Show that $\{s_n\}$ also converges to L.

5) Choose ONE to prove:

A) Suppose that $\{s_n\}$ is a sequence of real numbers and that $s_n \leq M$ for all $n \in N$, and that $\lim s_n = L$. Prove that $L \leq M$.

B) Show that if A and B are denumerable, then so is $A \times B$.

C) Show that $\lim(s_n + t_n) = \lim s_n + \lim t_n$, (assuming the two on the right exist).

BONUS: Prove