Notes on Strong Induction, MAA 3200, Fall 2010

We proved in class [10/5/10] that if Induction is valid, then Strong Induction is valid. The converse is also true, and the proof is much shorter, but a bit tricky. I will prove it here in an informal style, because I also want to explain the general concept of *strong*.

We say that $p \wedge q$ is a **stronger** statement than p, because $[p \wedge q] \Rightarrow p$. Likewise, $p \to r$ is stronger than $(p \wedge q) \to r$ (you can check that $[p \to r] \Rightarrow [(p \wedge q) \to r]$, from a truth table, or maybe by reasoning it out). This shows that weakening the premise (from $p \wedge q$ to p) strengthens the implication. Applying this **again**, we see that $[[(p \wedge q) \to r] \to s] \Rightarrow [[p \to r] \to s]$. Now, set

$p = p(n-1)$
$q = p(1) \wedge \ldots \wedge p(n-2)$
$r = p(n)$
$s = \forall n, p(n)$

Inserting these into the previous paragraph results in: Strong Induction implies Induction. [For simplicity, I omitted a couple of $\forall n$ phrases, and the basis step, but the idea is essentially correct].

In advanced math, you may see the words **Strong** and **Weak** used again, in various contexts. Think of them as mnemonic devices to help you remember which statements imply others. For example, there is a theorem [in graduate level real analysis] that if f_n converges (strongly) to f, then f_n also converges weakly to f. You really don't even need to understand what these phrases mean, to guess which one implies the other. Of course, the implication needs to be proven, but then the vocabulary helps us remember the result.