Fall 2003, MAA 3200, Key to HW 2

Here are answers to the 5 problems I chose to grade on HW2 (10 points each, + 50 general points). In hindsight, my grading may have been too gentle - the average was about 85. Feel free to ask about any other HW1 or HW2 problems you still aren't sure about.

Ch. 3.1-10a) They proved the converse of the statement ($Q \to P$).

10b) Assume $x \neq 4$ and $(2x-5)/(x-4) = 3$. Then, by algebra, $2x - 5 = 3(x - 4)$, etc, so $x = 12 - 5 = 7$. Done.

Ch.3.2-2. **Proof:** Assume $A \subseteq C$ and that $B$ and $C$ are disjoint (so $B \cap C = \emptyset$). Assume $x \in A$. (ets $x \notin B$). Since $A \subseteq C$, $x \in C$. Since $B \cap C = \emptyset$, we know that $x \in B$ and $x \in C$ cannot both be true. Since $x \in C$, we conclude $x \notin B$.

**Another Proof:** (almost the same, really). Assume $A \subseteq C$ and that $B$ and $C$ are disjoint (so $B \cap C = \emptyset$). Assume $x \in A$. (ets $x \notin B$). Assume $x \in B$ to get a contradiction. Since $A \subseteq C$, $x \in C$, so $x \in B \cap C$. This contradicts $B \cap C = \emptyset$. So, $x \notin B$. Done.

Ch.3.3-14a) Assume $a|b$ and $a|c$. So, by definition, there are integers $k$ and $m$ so that $ak = b$ and $am = c$. So, $b + c = ak + am = a(k + m)$. Since $k + m$ is an integer, $a|(b + c)$. Done.

Note: you should make the quantification of $k$ and $m$ clear (use a phrase like "there exists"). *Don't* call them both $k$. Be sure to mention that k, m and k+m are all integers.

Ch.3.4-2) Assume $A \subseteq B$ and $A \subseteq C$. Ets $A \subseteq B \cap C$. Let $x \in A$. Then by definition of $\subseteq$, $x \in B$ and $x \in C$. So, $x \in B \cap C$, as desired.

Note: The phrase "Let $x \in A$" is a shortcut, the standard reaction to the "Ets" above. You can also include phrases like "$\forall x, x \in A \to x \in B$" in your proofs, but using standard shortcuts will usually make your proofs clearer.

2nd Limit: I did this one in class. One common mistake was to use $N$

(which s/b used when $x \to \infty$) instead of $\delta$ (which is used in most other limit proofs).

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Here are a few more answers to misc HW1 and HW2 problems:

(1st HW2 Limit; not graded) State defn; then - Let $\epsilon > 0$. Set $\delta = \epsilon/3$. Let $x$ be arbitrary, and assume $0 < |x - 2| < \delta$. So, $|x - 2| < \epsilon/3$. [More Algebra Here] So, $|(3x + 1) - 7| < \epsilon$. Done.

Ch.1.3-3d. This says 4 is prime and that $13 - 8 > 1$. It has no free variables and is false.

Ch.1.5-4a. Draw a truth table, with columns labeled: p,q, $p \leftrightarrow q$ and finally $(p \wedge q) \vee (\neg p \wedge \neg q)$ (and any others as needed). The last column should match the one in Fig.6 (pg 50).

Ch.2.1-1a. Call the people x and y. Then, $\forall x, [\exists y, F(x, y)] \to S(x)$. Again, the parentheses are needed. I assumed that "y is a person" is understood, but it is OK to include a phrase like $y \in P$ or $p(y)$, if you want.