Here are answers to the 5 problems I chose to grade on HW2 (10 points each, + 50 general points). In hindsight, my grading may have been too gentle - the average was about 85. Feel free to ask about any other HW1 or HW2 problems you still aren’t sure about.

Ch. 3.1-10a) They proved the converse of the statement \((Q \rightarrow P)\).

10b) Assume \(x \neq 4\) and \((2x - 5)/(x - 4) = 3\). Then, by algebra, \(2x - 5 = 3(x - 4)\), etc, so \(x = 12 - 5 = 7\). Done.

Ch.3.2-2. Proof: Assume \(A \subseteq C\) and that \(B\) and \(C\) are disjoint (so \(B \cap C = \emptyset\)). Assume \(x \in A\). (ets \(x \not\in B\)). Since \(A \subseteq C\), \(x \in C\). Since \(B \cap C = \emptyset\), we know that \(x \in B\) and \(x \in C\) cannot both be true. Since \(x \in C\), we conclude \(x \not\in B\).

Another Proof: (almost the same, really). Assume \(A \subseteq C\) and that \(B\) and \(C\) are disjoint (so \(B \cap C = \emptyset\)). Assume \(x \in A\). (ets \(x \not\in B\)). Assume \(x \in B\) to get a contradiction. Since \(A \subseteq C\), \(x \in C\), so \(x \in B \cap C\). This contradicts \(B \cap C = \emptyset\). So, \(x \not\in B\). Done.

Ch.3.3-14a) Assume \(a | b\) and \(a | c\). So, by definition, there are integers \(k\) and \(m\) so that \(ak = b\) and \(am = c\). So, \(b + c = ak + am = a(k + m)\). Since \(k + m\) is an integer, \(a | (b + c)\). Done.

Note: you should make the quantification of \(k\) and \(m\) clear (use a phrase like “there exists”). Don’t call them both \(k\). Be sure to mention that \(k\), \(m\) and \(k+m\) are all integers.

Ch.3.4-2) Assume \(A \subseteq B\) and \(A \subseteq C\). Ets \(A \subseteq B \cap C\). Let \(x \in A\). Then by definition of \(\subseteq\), \(x \in B\) and \(x \in C\). So, \(x \in B \cap C\), as desired.

Note: The phrase “Let \(x \in A\)” is a shortcut, the standard reaction to the “Ets” above. You can also include phrases like “\(\forall x, x \in A \rightarrow x \in B\)” in your proofs, but using standard shortcuts will usually make your proofs clearer.

2nd Limit: I did this one in class. One common mistake was to use \(N\)
(which s/b used when \( x \to \infty \)) instead of \( \delta \) (which is used in most other limit proofs).

Here are a few more answers to misc HW1 and HW2 problems:

(1st HW2 Limit; not graded) State defn; then - Let \( \epsilon > 0 \). Set \( \delta = \epsilon / 3 \). Let \( x \) be arbitrary, and assume \( 0 < |x - 2| < \delta \). So, \( |x - 2| < \epsilon / 3 \). [More Algebra Here] So, \( |(3x + 1) - 7| < \epsilon \). Done.

Ch.1.3-3d. This says 4 is prime and that \( 13 - 8 > 1 \). It has no free variables and is false.

Ch.1.5-4a. Draw a truth table, with columns labeled: \( p, q, p \leftrightarrow q \) and finally \( (p \land q) \lor (\neg p \land \neg q) \) (and any others as needed). The last column should match the one in Fig.6 (pg 50).

Ch.2.1-1a. Call the people \( x \) and \( y \). Then, \( \forall x, [\exists y, F(x, y)] \rightarrow S(x) \). Again, the parentheses are needed. I assumed that “\( y \) is a person” is understood, but it is OK to include a phrase like \( y \in P \) or \( p(y) \), if you want.