MAA 3200, Key to HW 7

Here are answers to the problems I graded on HW7. Another 20 points for general care and completeness. The average was about 70/100.

2.4.3 [20pts] Assume $\{x_n\}$ is a Cauchy sequence in N. Let $\epsilon = 1/2$. So, $\exists N$ such that $m, n \geq N \rightarrow |x_m - x_n| < 1/2$. Since $|x_m - x_n|$ is an integer, this means $|x_m - x_n| = 0$ so that $x_m = x_n$. So, if $a = x_N$ and $n \geq N$, we get $a_n = a$ as desired.

2.5.1 [40pts] The liminf and limsup for each part are:

a) 2 and 4
b) -1 and 1
c) -1 and 1
d) 1/2 and 1/2
e) 0 and 0
f) 0 and $+\infty$
g) $+\infty$ and $+\infty$

Special 2abc [20pts]

2a) Done in class 11/26/03

2b) Define $[a][b] = [ab]$. So $3 \cdot 3 = [3][3] = [9] = [4] = 4$ (where the first and last equalities are just notational changes). Since $3 \cdot 2 = 1$ we see that 2 is the multiplicative inverse of 3.

2c) Z_6 does not have multiplicative inverses like Z_5 does. There is no multiplicative inverse of 3, because always 3x = 0 or 3x = 3. [The equation $2 \cdot 3 = 0$ also shows it is not field, by problem 1].

If there are any other problems you are not sure about, you should ask! Any of these could be on the final. For example, problem 2.4.1 about the sum of 2 Cauchy sequences should be pretty straightforward:

Proof: Assume $\{x_n\}$ and $\{y_n\}$ are Cauchy. Let $\epsilon > 0$. So, $\exists N_x$ such that $m, n \geq N_x \rightarrow |x_m - x_n| < \epsilon/2$. Likewise, $\exists N_y$ (etc). Assume $n \geq N = \max\{N_x, N_y\}$. Then by the triangle inequality, $|(x_m + y_m) - (x_n + y_n)| \leq |x_m - x_n| + |y_m - y_n| < \epsilon/2 + \epsilon/2 = \epsilon$. So, the sum is Cauchy. Done.