MAA 3200, Key to HW 1

Here are answers to 5 the problems I chose to grade on HW1 (10 points each, + 50 for overall completeness etc). I think the average will be about 70/100.

1.1.1a, pg12: $(r \vee h) \wedge \neg(h \wedge t)$

1.2.13, pg24: $P \vee \neg Q$ is the simplest answer, but anything equivalent is OK.

1.3.3a, pg 32 (5pts): x is bound. It says $19 > 1$ which is true.

1.3.3b, pg 32 (5pts): x is bound but c is free. It says $3 \leq c$.

2.2.5, pg 71: The idea of the hint is to relate $\exists$ to $\forall$, which we can do using $\neg$'s. The symmetry of the justifications below is rather interesting!

Proof: The following are all equivalent:
$\exists x(P(x) \vee Q(x))$ to
$\neg\neg\exists x(P(x) \vee Q(x))$ by double negation
$\neg\forall x\neg(P(x) \vee Q(x))$ by theorem on negation of quantifiers
$\neg\forall x(\neg P(x) \wedge \neg Q(x))$ by De Morgan's Law.
$\neg[\forall x\neg P(x) \wedge \forall x\neg Q(x)]$ by the fact in the hint
$\neg\forall x\neg P(x) \vee \neg\forall x\neg Q(x)$ by De Morgan's Law.
$\exists x, \neg\neg P(x) \vee \exists x\neg\neg Q(x)$ by theorem on negation of quantifiers
$\exists x, P(x) \vee \exists x Q(x)$ double negation (Done).

2.3.7, pg 80: If you tried this, you got it. Almost any pair of sets will work. For example, set $A = \{a\}$ and $B = \{b\}$. You can check that $P(A) \cup P(B)$ contains 3 sets, but $P(A \cup B)$ has 4 sets.