MAA 3200 HW on Number Systems; Fall 2018

These are fairly simple questions based on my online file about N, Z and Q, and perhaps also on the lectures from 10/2/18 and 10/4/18. Feel free to ask about anything that is not clear. Or you can look at the Morash book, or perhaps Kane's book, if you need more details.

1) Short Answer Questions:

a) Which famous proof method is used the most here, especially for properties of N ?
b) Why are the proofs about Z and Q usually shorter than those for N ?
c) How does Q differ from Z ? [Which operation(s), partial order(s), etc, are defined in Q but not Z]
d) Which of our four main number systems (N,Z,Q and R) are fields ? (maybe more than one)
e) What is a binary operation on a set A ?
f) What does *well-defined* mean, for example when defining an operation on equivalence classes ?
g) Which equivalence relation was used to define Q ?

2) Find these elements of N:

a) $\sigma(4)$
b) $s_3(2)$
c) $s_3(\sigma(2))$ and $\sigma(s_3(2))$ - are they equal?

3) Use the definition of $<$ in N to show that $3 < 10$. This isn't supposed to be hard; you can assume that 1, 3, 4, 7, 9 and 10 are in N, and you can use simple formulas like $1 + 3 = 4$ without proof.

4) Prove that every nonzero element of Q (as defined in class and in the online file) has a multiplicative inverse. You should suggest an equivalence class and justify that it works.

5) Before defining $+$ on N, we defined (for each fixed $m \in N$) a function $s_m : N \to N$ by:

a) $s_m(0) = m$

b) $s_m(\sigma(n)) = \sigma(s_m(n))$

Then, we defined $m+n$ as $s_m(n)$. Now, we can prove the commutative property, $m+n = n+m$, by induction on n. For your HW, just prove the basis step; set $n = 0$), and ETS $\forall m \in N,\ 0+m = m+0$.

Hints: apply the definition of $+$, to rephrase the ETS: $\forall m \in N,\ s_0(m) = s_m(0)$. Part a) further simplifies one side, so ETS: $\forall m \in N,\ s_0(m) = m$. Now, use induction on m, and use (a) and (b) as needed. [I plan to prove that s_m exists in class. If so, you may use formulas from my proof in your proof, if you state where they come from].

6) Prove trichotomy for $<$ on Z, using trichotomy for $<$ on N. Of course, you will need to use the definition of $<$ on Z and may have to think about what $=$ means for Z. Your proof will probably use cases.