MAD2104
Suggested problems on Chapter 1 material
(Logic)

1. Let p and q be the following propositions.
p: It is below freezing.
q: It is snowing.
Write the following propositions using p and q and logical connectives (including negations).
a. It is below freezing and snowing.
b. It is below freezing but not snowing.
c. It is not below freezing and it is not snowing.
d. It is either snowing or below freezing (or both).
e. If it is below freezing, it is also snowing.
f. Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.
g. That it is below freezing is necessary and sufficient for it to be snowing.

2. Let p, q, and r be the following propositions.
p: Grizzly bears have been seen in the area.
q: Hiking is safe on the trail.
r: Berries are ripe along the trail.
Write the following propositions using p, q, and r and logical connectives (including negations).

a. Berries are ripe along the trail, but grizzly bears have not been seen in the area.

b. Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.

c. If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.

d. It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.

e. For hiking on the trail to be safe, it is necessary but not sufficient that berries not be ripe along the trail and for grizzly bears not to have been seen in the area.

f. Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail.

3. Construct a truth table for each of these compound propositions.

a. $p \to \neg p$

b. $p \leftrightarrow \neg p$

c. $p \oplus (p \vee q)$

d. $(p \wedge q) \to (p \vee q)$

e. $(p \vee q) \to (p \oplus q)$

f. $((p \to q) \wedge (q \to r)) \to (p \to r)$

g. $(p \oplus q) \oplus (p \oplus \neg q)$

4. Show that each of these conditional statements is a tautology by using truth tables.

a. $(p \wedge q) \rightarrow p$
b. $p \rightarrow (p \vee q)$
c. $\neg p \rightarrow (p \rightarrow q)$
d. $(p \wedge q) \rightarrow (p \rightarrow q)$
e. $\neg(p \rightarrow q) \rightarrow p$
f. $\neg(p \rightarrow q) \rightarrow \neg q$

5. Show that each conditional statement in Question 4 is a tautology **without** using truth tables.

6. Let $P(x)$ denote the statement "$x \leq 4$." What are these truth values?
a. $P(0)$
b. $P(4)$
c. $P(6)$

7. Let $Q(x, y)$ denote the statement "x is the capital of y." What are these truth values?

a. $Q(\text{Denver}, \text{Colorado})$

b. $Q(\text{Detroit}, \text{Michigan})$

c. $Q(\text{Massachusetts}, \text{Boston})$

d. $Q(\text{New York}, \text{New York})$

8. Translate these statements into English, where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny" and the domain consists of all people.
a. $\forall x(C(x) \to F(x))$
b. $\forall x(C(x) \land F(x))$
c. $\exists x(C(x) \to F(x))$
d. $\exists x(C(x) \land F(x))$

9. Let $C(x)$ be the statement "x has a cat," let $D(x)$ be the statement "x has a dog," and let $F(x)$ be the statement "x has a ferret." Express each of these statements in terms of $C(x)$, $D(x)$, $F(x)$, quantifiers, and logical connectives. Let the domain consist of all students in your class.
a. A student in your class has a cat, a dog, and a ferret.
b. All students in your class have a cat, a dog, or a ferret.
c. Some student in your class has a cat and a ferret, but not a dog.
d. No student in your class has a cat, a dog, and a ferret.
e. For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.

10. Let $P(x)$ be the statement "$x = x^2$." If the domain consists of the integers, what are these truth values?
a. $P(0)$
b. $P(1)$
c. $P(2)$
d. $P(-1)$
e. $\exists x P(x)$
f. $\forall x P(x)$

11. Determine the truth value of each of these statements if the domain consists of all integers.

a. $\forall n(n + 1 > n)$
b. $\exists n(2n = 3n)$
c. $\exists n(n = -n)$
d. $\forall n(3n \leq 4n)$

12. Determine the truth value of each of these statements if the domain consists of all real numbers.
a. $\exists x(x^3 = -1)$
b. $\exists x(x^4 < x^2)$
c. $\forall x((-x)^2 = x^2$
d. $\forall x(2x > x)$

13. Determine the truth value of each of these statements if the domain consists of all integers.
a. $\forall n(n^2 \geq 0)$
b. $\exists n(n^2 = 2)$
c. $\forall n(n^2 \geq n)$
d. $\exists n(n^2 < 0)$